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Shareholding strategies for selling green products on online platforms in a two-echelon supply chain

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Abstract

Observing the practical vertical shareholding phenomena in platform retailing, this paper considers two shareholding rates (forward and backward) and proposes three shareholding strategies (forward/backward and cross-shareholding) in a supplier-lead green supply chain to investigate the operation mechanism behind it. Shareholding's impacts on green investment, prices and profits are provided and the players' strategy preferences are discussed. We interestingly find that cross-shareholding can be joint optimal when the two shareholding rates are both sufficiently low and thereby Pareto region goes with it. Meanwhile, both the players' performances can be further improved through certain coordinated ways.

Keywords: Green supply chain, Game theory, Platform retailing, Shareholding strategy, Pareto region

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1. Introduction

Platform retailing has been growing rapidly for the past few years. In 2017, the transaction scale of platform retail market in China increased 39.2% compared to the previous year and reached 7,175 billion yuan[1], and this number is expected to reach 1.7 trillion by 2020[2]. In general, this fast-growing platform retailing can be devided into agency selling scheme and reselling scheme based on whether the supplier can access to the customers directly (Abhishek et al., 2016; Kward et al., 2017). In agency selling scheme, the retailer allows its supplier(s) to sell products on its platform (e.g., Apple on TMall) for a commission fee or part of the revenue (Shen, 2018; Zhang et al., 2019). In reselling scheme, the online retailer purchases products from manufacturers or suppliers and then resells them to consumers (e.g., JD.com sells Apple's products). Although agency selling is fairly popular, reselling still remains vitality in the platform retailing field. For example, JD.com, which was a pure retailer in its early stage (Chen et al., 2014), still gains a huge portion of revenue through reselling business now (i.e.,

[1] http://www.stats.gov.cn/, National Bureau of Statistics of China

[2] https://www.chinainternetwatch.com/19978/retail-jan-feb-2017/, China online retail grew by 31.9% to exceed US$124 bn in Jan & Feb 2017

JD self-run, around 90% in 2019)[3]. Similarly, although Amazon's reselling business has shrunk to a relatively small scale (Jiang et al., 2011), it still exists and continually creates considerable profits for the company (Chen et al., 2014).

No matter under which kind of selling scheme, the upstream supplier (or manufacturer) will invest in various sides to attract the final customers, among which green investment is particularly noteworthy nowadays (Hong et al., 2018). This investment has become one of the industry's foremost strategic challenges from engineering a global concern, business and marketing perspective (Li et al., 2019a,b) and proved to be profitable (Geng et al., 2017; Fahimnia et al., 2018). Driven by this pecuniary benefit, on the one hand, many manufacturers have spent much money on the R&D of green products, such as GE's $432 million investment project for green refrigerators[4] and Gree's green renewable energy companies in Zhengzhou and Zhuhai[5]. These also appear in other industries like intelligent hardware (Apple), clothing (Converse) and drink (Coca-Cola, Dong et al. 2019). Specifically, Apple has invested in building a closed-loop supply chain and lowering the energy cost to improve the greenness of its products as well as its whole supply chain. The renewable aluminum, tin and rare-earth are 100% used in its products, and its most packages like fiber shopping bag are also made from renewable materials[6]. These all formed a green corporate image and forced consumer's trust and loyalty. Similarly, Converse's investment in Renew project on upcycled textiles, recycled PET and cotton canvas blends[7] and Coca-Cola's expenditure in the plastic packaging made from plants[8] also cost much but improve both their consumer images and market performances. On the other hand, the platform retail industry also attaches great importance to green products. However, they seldom pay for the green investment of their suppliers. For example, in the first half of 2017, JD.com's green brand growth rate rose to 16.1% (Du et al. 2019) and JD.com only provided a more convenient channel instead of substantial benefits for its suppliers. Moreover, there even exist certain e-stores such as EGP[9] specialize in green products, which just operates as a common platform and enjoys the greenness of its suppliers freely.

Such greenness-driven phenomena bring free-riding problems which may weaken the green investors' initiatives and lead to under-investment issues. That is, as an upstream manufacturer invests in green products, its downstream retailer will get a "free riding" in the sense of gaining more profits through a higher sales volume or price without other expenditures. This may of

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[3]https://ir.jd.com/annual-reports JD.com 2019 Annual Report

[4]https://pressroom.geappliances.com/news/ge-to-invest-432-million-create-187073, GE to Invest $432 Million, Create 500 Jobs, to Establish U.S. Centers of Excellence for Design and Manufacture of Refrigeration Products

[5]Gree annual report of 2019

[6]https://www.apple.com.cn/

[7]Three Ways Converse's Renew Initiative is Rethinking Waste. https://news.nike.com/news/converse-sustainable-chuck-taylor-renew-initiative

[8]The Coca-Cola Company Accelerates Global Production of Plastic Packaging Made from Plants. https://www.coca-colacompany.com/press-releases/coca-cola-accelerates-production-of-plastic-packaging-from-plants

[9]www.environmentalgreenproducts.com/store/home.php, Environmental Green Products

course reduce the motivation of the former and lead to inefficiency (Höffler and kranz, 2011). Although agency selling scheme could solve this problem to a large extent, it may subject to the irreconcilable bargaining of participants or the intricate settings. For instance, Gree and Suning failed to continue the agency selling scheme and broke up in 2004. Under the reselling scheme, some players often adopt different contracts proactively. However, as contracts are various, they may be limited by the complicated procedures and defaults risks. Facing this challenge, companies can resort to passive methods like the ready-made financial tools to seek common interests and alleviate the free-riding problem.

As a ready-made financial tool, holding share, allowing the shareholders to take partial profit of the target company, can be implemented to reduce the free-riding problem spontaneously without bargaining progress. Specifically, one company can purchase certain portion of another one's share to benefit from its operation, and the target firm may also benefit from this progress through acquirer's incentive that making the target firm better. Generally, there exist two dimensions—horizontal shareholding and vertical one in this tool, both of which can be witnessed in the practical economic activities. However, in the retailing industry, while B&Ms (Brick and Mortars) incline to horizontal holding share (e.g., CR Vanguard[10]) or even acquisition (e.g., CR Vanguard[11], Carrefour[12]), the online platforms are also delighted to choose vertical ones expecting the horizontal shareholdings. This is because the B&Ms often face smaller market shares compared with the online platforms (i.e., the top three B&M retailers' total market share is 3.05% while the platform retailers' is 52%), and thus vertical cooperation methods like shareholding are not so profitable for them.

More specifically, the vertical shareholding can be divided into three categories as forward, backward and cross-shareholding. For example, MengNiu Dairy (a dairy manufacturer) cooperates with Alibaba (a platform retailer) and invests in a co-established milk retailing enterprise Tianxianpei[13] in the form of forward shareholding. Meanwhile, Alibaba's holding 25.25% shares of Haier multimedia[14] and Suning's acquisition of TCL's partial shares[15] showing the prevalent of backward shareholding. As a combination of both strategies, cross-shareholding is also widely adopted and believed can boost the greenness as well increase the competitiveness of supply chain[16]. A practical example can be witnessed in Gree and its major retailers. Gree, broke up with Suning in agency selling and turned to buy partial shares of its retailers. Meanwhile, some of these retailers combine together to form a new company Jinghai and purchased part of Gree's share. This shareholding structure simulates Gree to invest more in greenness of the products

[10] CR Vangurad 2017 annual report

[11] Govt OKs CRE's merger with Tesco http://en.ce.cn/Business/Enterprise/201406/01/t20140601_2907189.shtml

[12] Carrefour selling 80% equity interest to Suning.com http://www.ecns.cn/business/2019-06-24/detail-ifzkmhnc3006877.shtml

[13] https://www.txp-life.com/

[14] Annual report of the company

[15] TCLTechnology Group Corporation 2019 annual report

[16] Shanxi province sees tech as key to its industrial transformation. http://www.chinadaily.com.cn/m/shanxi/2018-04/17/content_36155399.htm

(e.g., established green reproducible resources subsidiaries) and helps Gree become one of the most competitive air-condition brands in the Chinese market (it's online market share reached 28.45% in the first three quarters in 2020). Observing these shareholding practice, whether and how to use a shareholding strategy as well as its profitability worth in-depth study.

The benefits and constrains behind these vertical shareholdings are varied in the realistic. Overall, forward shareholding allows the upstream firm to access the final market more directly and improve its sales ability, which also increases its motivation to provide better products for the downstream firm. While backward shareholding can bring the downstream firm more reasonable price, it exerts an incentive for the downstream firm to order more and thus benefits the upstream one. However, the target firm will loss parts of its share and may finally incur losses either in the forward and backward shareholding. Through cross-shareholding, the two companies will hold partial shares of each other and both mentioned benefits and constrains may exist. Furthermore, as a combination of two single-way shareholdings, both shareholding behaviors will interact with each other and thereby influence the performances of the players.

Motivated by the aforementioned discussion, the study reported in this paper investigates three interrelated issues as follows. First, how will the two shareholding rates under different shareholding strategies affect the optimal decisions of the supply chain including green level, wholesale price and retail price? Second, how does shareholding influence the profits of the players, and which shareholding strategy may be chosen by each party of a supply chain? Combining both players' preferences, does a Pareto-region exist? Third, from the perspective of the entire chain, which shareholding strategy is more profitable? Does it exist any mechanism to further increase shareholding level and achieve higher chain profits without damaging any players? If it exists, how to adopt it?

To address these questions, we model a Stackelberg game in a green supply chain consisting of one supplier (leader) and one platform retailer (follower). Having green-sensitive consumers on the sales terminal and acting as the Stackelberg leader, the supplier invests in the greenness of the products to boost the final demand but increases its cost conversely, which favors the retailer at no cost and lead the under-investment of the supplier. To alleviate this problem, we propose a novel scheme—shareholding including three different shareholding strategies in this chain based on a traditional reselling scheme. In particular, under the forward shareholding strategy, the upstream firm holds part of the downstream firm's shares and the downstream firm holds the upstream firm's shares under the backward shareholding strategy. Both of these characteristics are combined under cross-shareholding strategy. Although holding parts shares of a firm cannot allow a shareholder to take part in the held firm's decisions directly, it does play a vital importance role in the shareholder or the held firm's total profit and the mechanism behind is complicated and indirect. Specifically, while shareholding can influence the decisions of both the shareholder and held firm, holding another firm's share may result a loss and being held by others may lead to higher profits. In this paper, the impacts of different shareholding strategies are revealed and certain unexpected results are proved thereupon. We supplement the existing literature by combining shareholding with green investment and comparing different shareholding strategy rather than focusing one single strategy. As a whole, the main findings of this work comprise the following three aspects.

Firstly, we find that the optimal green level is positively associated with both shareholding rates in all shareholding strategies, and is more sensitive to backward shareholding rate rather than the forward one. These observations show that, a closer relationship of both players can lead to a higher green investment of the products. The mechanism behind it derives from the dominant position of the supplier, which allows it to benefit much more than the retailer from a higher green level at any shareholding strategies. Moreover, although it naturally reaches that higher forward shareholding rate favors the green investment, the backward shareholding rate can also improve it. Because this shareholding can incentivize the retailer to order more and pay higher, which thereby provides motivation to the supplier to invest in the products. This is even more effective than the direct inducement in forward shareholding, highlighting the passive influence of the shareholding. Regarding pricing behaviors of the supply chain participants, in most cases, both the wholesale price and retail price increase with the backward shareholding rate and decrease with the forward one. However, counterintuitively, the wholesale price may rise with respective to the forward shareholding rate on certain conditions under cross-shareholding strategy. In other words, while changing the forward shareholding rate in the cross-shareholding strategy, the wholesale price and retail price may change in different directions. It indicates that the supplier becomes more powerful under this structure, and thus will shrink the platform retailer's margin when holding a sufficiently large scale of the platform retailer's shares.

Secondly, we discuss the strategy preferences of both the supplier and the platform retailer. Specifically, the upstream supplier prefers the cross-shareholding strategy, which is mainly due to its leader power. That is, through adjusting its wholesale price and green level, the supplier can always take advantage of better coordination origin in higher shareholding rates. Whereas, it's not always true for the platform retailer as it prefers forward shareholding on most occasions. This is because the aggregate benefits of the low wholesale price and high green level while employing forward shareholding strategy help cover the loss in shares in forward shareholding, that is, the passive incentive benefit exceeds the be-held share loss. As the follower of the chain, the benefits from initiative behavior are highly subject to the supplier and the incentive effects (passively) play a more important role. Nonetheless, when the forward shareholding rate exceeds a certain threshold and backward shareholding rate is low, the loss in shares combined with the shrinking margin damages the platform retailer and backward shareholding becomes more profitable. If both shareholding rates are extremely high, none of the three shareholding strategies is beneficial to the platform retailer. Particularly, when two shareholding rates are low, both the supplier and platform retailer can gain the most profits through cross-shareholding strategy simultaneously, and Pareto-region goes with it. This observation provides some evidence to support the extensive cross-shareholding phenomena and their low levels in the Japan market (its average cross-shareholding rate is around 9%, NRI, 2019). Facing the additional choice of agency selling, we observe the shareholding remains the optimal for both players under certain circumstance.

Thirdly, for the whole supply chain's performance, we conclude that the impacts of both shareholding rates are not monotonous. Unlike a higher forward shareholding rate always leads to a higher profit, increasing the backward shareholding rate may hurt the chain after a point. Therefore, cross-shareholding is the optimal shareholding strategy in the view of the whole chain. Besides, we observe that the turning point of the retailer's profits is smaller than the one of whole supply chain. Owing to this difference, all the participants combines with the whole supply chain can possess better performances if the supplier can compensate the retailer's being held or holding other's share. This observation provides a method in improving the performances of the players that is unique in shareholding, which is similar to the practice that some huge enterprises like Haier and Luzhou Laojiao provide their virtual shares (e.g., in the form of option) freely for retailers to incentivize them. The robustness of our conclusions is checked by adopting specific kind of endogenous share prices and considering another term of green investment cost function, respectively.

These above findings have important implications for company managers as well as the regulator. First, as shareholding can promote the green level of a supply chain, the governor can consider to encourage it in vertical related supply chains for green/environmental goal. In the mean time, as green level is more sensitive to the backward shareholding, encouraging the backward shareholding is more effective than increasing the forward one. Second, for the manager of the supplier, she/he can try to increase the level of the holding shares rate and also the being holding shares rate. For the retailer's manager, she/he should avoid being held too much shares or holding a high level of the supplier's share even it's costless. Combining with their strategy choices, low level cross-shareholding is the most achieveable when their managers doing the corresponding decisions. Third, considering the performance of the whole supply chain, through appropriate adjustment on the share transactions, the upper limits of both shareholding rates can be promoted and both the supplier and the retailer's performances can be improved. At the same time, as a higher shareholding rate does not always lead to a better performance (i.e., the chain's profit is concave to the backward shareholding rate), a supply chain should avoid blindly pursuing interest uniformity of the supply chain members through setting too high shareholding rates.

The remainder of this study is arranged as follows. In Section 2, we do a brief literature review on platform retailer or online retailing, green supply chain management, and shareholding strategies in vertically related chain. Then we establish our model and give out the traditional reselling benchmark model and three shareholding strategies in Section 3. In Sections 4, we compare the optimal solutions achieved using the proposed model. Section 5 contrasts the profits obtained with different shareholding strategies. The shareholding strategy preferences of the participants are also discussed in this section. Section 6 explores shareholding's impacts on the entire supply chain. In the extension part of Section 7, we consider the existence of agency selling, share price and extend our model to another form of green investment cost, respectively. In the last section, we conclude this study and propose its managerial implications as well as future research aspects. All of the proofs and supporting contents are presented in Appendix.

2. Literature Review

As discussed above, three main streams of literature are particularly related to the present research: (i) platform retailing or online retailing, (ii) green supply chain management and (iii) forward, backward and cross-shareholdings in the vertically related supply chain. These are briefly reviewed below.

2.1. Platform retailing

Platform retailing plays important roles in improving supply chain performance through operational innovation and attracts great attentions in the recent years. Jiang et al. (2011) used a hybrid model of Amazon as both a reseller and a platform to explain the existence online platform retailing and prove its profitable. Thereby, two main streams of studies arose in this area, one focuses on whether to introduce the agency selling (Jiang et al., 2011; Yan et al., 2019; Zhen et al., 2020) and another discusses the choice between agency selling and reselling (Abhishek et al. 2016; Kward et al. 2017).

For the introduction of agency selling, Jiang et al. (2016) investigated whether peer-to-peer (P2P) marketplaces benefit traditional supply chains when consumers may experience valuation risk in a two period model setting. Yan et al. (2019) claimed that marketplace channel should be introduced under both low degree and high degree of upstream sales inefficiency, which also means that a weak direct channel would not necessarily become a burden.

For the choice of these two schemes, Abhishek et al. (2016) examined when should the online retailers use an agency selling format instead of using the more conventional reselling format in a supply chain consisting one supplier and two online retailers. They find that online retailers prefer agency selling when sales in the electronic channel lead to a negative effect on demand in the traditional channel. Wei et al. (2020) also addressed this issue by considering the combined effects of manufacturers' leader-follower relationships, the e-tailer's referral fees, the difference in products' substitutable degrees and the difference in products' market bases. Shen et al. (2019) studied the interaction of a dominant online retailer's decision on the selling format and a manufacturer's decision on the channel selection and find that the demand substitution between the dominant retailer and the weak retailer plays an important role in determining channel selection. Zhang et al. (2019) focused on the interrelationship between a platform's contract choice and a manufacturer's quality decision and find if quality is exogenous, the platform will always adopt revenue sharing.

Our work is also particularly relative to the work of green investment of the platform supply chain. Among the related literature, Du et al. (2019) studied the platform-led green advertising problem and demonstrated the value of green advertising from both economic and environmental perspectives. The readers can refer to other works which discussed this issue in the aspects like consumers welfare Johnson (2016), competition effect (Shen 2018; Zennyo 2020), information asymmetry (Kward et al. 2017; Xiao and Xu 2018), and new-tech (Choi 2019).

Different from the above works on the introduce of agency selling or the comparison of these two schemes, we focus on adopting a novel mechanism—shareholding based on the traditional platform reselling. While both the agency selling and contract schemes are proactive methods, shareholding that proposed in this paper can be seen as a passive way, which reasonablely gives out some new insights. We prove that shareholding scheme can be accepted by both supply members and emphasize the shareholding rates' impacts on the decisions and performances of the players under platform retailing. Besides, rather than a downstream-led green investment in the established work, upstream investment is considered in our model, and through which we supplement the existing literature.

2.2. Green supply chain management

Green supply chain management has been a hot topic in recent years. Our paper is related to the contracting or other mechanisms in the green manufacturing (Liu et al., 2020), and among them sharing contracts are the most prevalent. They are utilized to improve the supply chain performance, which can be divided into cost-sharing contracts and revenue-sharing contracts (Adhikari and Bisi 2020). Regarding the cost-sharing contracts, Ghosh and Shah (2012) explored the problem in a supply chain involving green-sensitive customers and analyzed the cost-sharing contract's impacts concerning the decisions of the interested parties, showing that cost-sharing contracts lead to a higher green level and higher profits. They subsequently extended the model by applying bargaining in the cost-sharing contract, demonstrating that bargaining is beneficial to the green level and thus, it helps increase supply chain profit (Ghosh and Shah, 2015). Xu et al. (2020) considered the cost sharing of green investment under greenness competition. They prove that the advantage of cost sharing is gradually weakened with the intensification of greenness competition.

Regarding revenue-sharing contracts, Giovanni (2014) analyzed the coordination problem of a green supply chain and found that the greenness level could be effectively improved, achieving Pareto-improvement. However, in a shareholding model, whether the wholesale price is higher or lower than the complete independence scenario is dependent upon the shareholding rates and types. Note that there also exist approaches for extending this topic by considering more complicated properties of the supply chain, such as governance (Hafezalkotob 2017; Song et al. 2017; Gao et al. 2018; Huang et al. 2020), information (Zhang et al. 2018; Choi and Luo 2019; Hong et al. 2018), multi-type products or competition (Liu et al. 2012; Zhu and He 2016; Guo et al. 2020; Wang et al. 2020).

The above outline focused on the contracts or complex structures of the chain and seldom considered the financial instruments especially shareholding between the firms in this field. Yet, upstream green investment combining with vertical shareholdings are very common in real world and needs to be explored. Based on our discussion, we extend this stream of literature by exploring the shareholding strategy's effect upon a green supply chain comprising one supplier (manufacturer) and one platform retailer in this paper, and show that the shareholding strategies and shareholding rates have great effect on the green investment. Moreover, we also propose that the indirect benefits under shareholding are more efficient than the direct ones in the improving of green investment.

2.3. Shareholding strategies in supply chain

The literature is abundant concerning the field of shareholding in supply chain. Based on typical industrial practice and existing literature, approaches to shareholding in vertical related supply chain can be grouped into three categories: forward, backward and cross-shareholding.

2.3.1. Forward and backward shareholdings

Forward and backward shareholding both are forms of single-way shareholding, having each gained significant attention from recently. This work focuses on the pricing or investment behavior of the supply chain players involving shareholdings. In relevance to such studies, Letizia and Hendrikse (2016) showed that as the forward shareholding rate increases, the supplier's incentive to invest becomes higher. This observation echoes the finding of the present analysis, which also indicates that increasing the backward shareholding rate will also boost the investment of the upstream firm. In particular, regarding the pricing behavior, Greenlee and Raskovich (2006) discussed the backward shareholding in a supply chain with multiple downstream firms. With the increase in backward ownership, the upstream firm will raise its price. Similarly, Hunold and Stahl (2016) considered a passive backward shareholding strategy in a two-supplier and two-retailer supply chain, discovering that if the competition level is high, passive backward ownership leads to increased downstream price and thus is strictly anticompetitive. Further, Hunold (2020) observed that the this shareholding may lead to entry deterrence. The present work in this paper confirms that backward shareholding will increase the wholesale price of the supplier and forward shareholding will decrease the price of the supply chain. However, we observe that when both shareholdings co-exist, that is, under the so-called cross-shareholding scenario, the latter conclusion may not always be true. Note that regarding the order quantity/output of a supply chain, Greenlee and Raskovich (2006) claimed that demand will be prompted the backward ownership. Similarly, Höffler and kranz (2011) observed that the increase of the incumbent's ownership share will lead to a higher order quantity. Nevertheless, running the proposed model herein reveals that backward shareholding will increase the final price and thus hurt the sales volume. Regarding performance of the players, Concerning the performance of the players, Gaigné et al. (2018) proved their benefits could be quite significant through applying forward acquisitions. In an N-to-One assembly system, Fu et al. (2018) considered both upstream and downstream Stackelberg game, concluding that such a shareholding system enables the downstream firm to share the profit of the upper ones and the latter to gain needed resources, which in turn eventually benefits the whole supply chain. However, we find that it may damage the platform retailer on most occasions. Indeed, we also observe that the backward shareholding may hurt the entire supply chain when the shareholding rate is sufficiently high. There are other studies that exploit heuristics into shareholding, such as that reported in Flath (1989), which reflects the relationship between shareholding and vertical integration. Qi et al. (2015) dealt with whether invested shares may be utilized for a competitor with a spillover effect.

Unlike the aforementioned studies that focus on one single shareholding strategy, we compare these two shareholdings herein to discuss the differences of them in a same framework. Moreover, we consider that the upstream firm can invest in the greenness of a product to raise the terminal sales volume, which also arises some new insights in this mechanism's influence on the green investment. We also consider the cross-shareholding scenario that both forward and backward shareholdings co-exist at the same time. This investigation will lead to amendment of certain existing conclusions on shareholding rates as discussed in the above review.

2.3.2. Cross shareholding

Cross-shareholding, which allows two (or more) companies to join together through a two (or more) sides shareholding agreement, can enhance the cooperation of different parties and help to achieve a win-win result. Driven by this motivation, Chen et al. (2016) studied the effect of partial cross-shareholding in push and pull supply chains, finding that the profit of players is independent of the leader's shares held by the follower. Indeed, when the leader increases its shareholding of the follower, the profit of the follower does not necessarily increase or decrease. Their results seem to indicate that a win-win result would only be achieved given certain appropriate settings. In our study, similarly, we also find that the leader holding the shares of the follower may either either lead to an increase or decrease of the follower's profit. Different from Chen et al. (2016), however, this work reveals that either type of shareholding would lead to a higher profit for the supplier because of its leadership in the supply chain, through which the supplier can adjust its green level and transfer the loss in shares to the platform retailer through a higher wholesale price. Güth et al. (2007) compared the seller's and buyer's attitudes to cross-shareholding and proved that applying this scheme might be sufficient to obtain more profits for both parties. They also suggested that buyers would be more likely to agree on cross-shareholding than sellers. However, our work leads to opposite conclusions in that the seller is more inclined to cross-shareholding and that the buyer may even end up with a loss when the cross-shareholding rates are high. In the recent literature, Fu and Ma (2019) considered cross-shareholding in a push and pull supply chain, demonstrating that both the upstream firm's shareholding and downstream firm's shareholding can increase the entire supply chain profit under certain contracts. They also showed that the win-win coordination could only be achieved when one firm would first operate at a loss. Our work reveals that cross-shareholding often benefits the whole supply chain. Nonetheless, the profit of the supply chain will decrease to the downstream firm's shareholding rate if the rate exceeds a certain threshold.

In short, the above literature review shows that academic research on shareholding is plentiful. However, existing works seldom consider drawing shareholding mechanisms into the investment model, especially when a green supply chain is addressed, leading to the knowledge gap between the literature and practical. To the best of our knowledge, this study is the first to discuss these two issues in this field: 1) unlike the traditional research's focusing on the pricing, ordering behaviors and profits, shareholding's impacts on green investment are investigated; and 2) rather than focus on one shareholding strategy like the existing literature, the impacts of forward, backward or cross-shareholding upon the players' decisions and performances are compared together, thereby deriving practically helpful novel conclusions.

3. Model Formulation and Schemes Presentation

3.1. Model establishment

Consider a green supply chain with one supplier and one platform retailer that competing each other in a classic Stackelberg game where the supplier acts as the leader and the platform retailer as the follower. This supply chain faces green-sensitive consumers who will buy more units of products at a higher level of greenness (Bearse et al., 2009), leading to a greenness related demand as

$$d = A - bp + a\theta,$$

where A is the original market size, b and a are the consumers' demand sensitivities to the price and green level, respectively (Li et al., 2018, 2019c; Hong et al., 2020). Therefore, to gain more profits from this greenness-related demand expansion, the supplier will invest in the greenness of the products at a cost of $\frac{1}{2}\beta\theta^2$ (Guo et al. 2020; Hong et al. 2020; Xu et al. 2020) in addition to a fixed unit cost c (Shen et al. 2019), where β ($\beta > 0$) represents the parameter of investment costs and θ denotes the green level of the products (Zhang et al. 2018; Guo et al. 2020) It is reasonably assumed that the greatest demand of the market is non-negative without considering greenness, that is $A - bc > 0$. In addition, to ensure the investment in green technology is higher than the marginal environment-related demand increase (Hong and Guo 2019), it is assumed that $\frac{1}{2}\beta > \frac{a^2}{b}$ (i.e., $\frac{1}{2}b\beta > a^2$), as commonly presumed in the abundant green supply chain literature. Obviously, if the supplier invests in the greenness of the products, the platform retailer will earn more profits without any extra costs, which can be seen as a free-riding and will hurt the motivation of the supplier. To resolve this problem, an effective method—shareholding can be considered to be adopted in this kind of supply chain with a large provider and a selling platform (e.g., Gree and Jinghai), which impels either or both parties of the chain to seek identical benefit passively.

In this vertically related supply chain, the two firms can herein hold each other's shares in different shareholding strategies comprising of the forward, backward and cross-shareholding strategy. To characterize different structures of shareholding strategies, both the forward and backward shareholding rates are considered. In particular, let k denote the forward shareholding rate and j the backward one. Thus, the supplier holds $1 - k$ share (in terms of rate) of its own company and j share of the platform retailer ($k, j \in [0,\ 1]$), while the platform retailer holds the remaining shares. If $k > 0$ and $j = 0$ (or $j > 0$ and $k = 0$), the relationship between the two parties becomes pure forward shareholding (or backward shareholding). If k and j are both non-zero, the two parts of the supply chain behave as cross-shareholding. To guarantee the decision-making independence of both firms, k and j should be smaller than 0.5 (Güth et al. 2007), otherwise (i.e., $k > 0.5$ or $j > 0.5$) the platform retailer/supplier will lose its decision power and become a subsidiary firm of the supplier/platform retailer and can be seen as a vertical merger (Lan et al. 2019). To show the shareholding strategies more clearly, Figure 1 (no shareholding) and Figure 2 (forward shareholding) are provided (to conserve space, backward and cross shareholding's diagrams are placed in the appendix).

Based on our descriptions above, we decompose the impacts of the shareholding on both players into two aspects, holding share effects and incentive effects, which are also the essential trade-off in the mentioned three shareholding strategies. On the one hand, holding share effects are naturally to be described as it allows a firm to share part of another one's profit while the latter suffers a loss. On the other hand, for the incentive effects, which are exclusive in the related supply chain members with shareholding, impels the shareholder to operate more favorable to the be held company for higher holding share effects and conversely brings the latter interests. Note however that both the supplier and the platform retailer may achieve higher or lower share pies under various shareholding structures, the incentive effects may alter the final achievements of their whole profits. Thereby, the analysis and discussion in the rest of this paper are basically based on these two effects.

Two issues are to be elaborated on the shareholding models in this paper. First, the transaction period of share and its price are not considered. As we focus on the shareholding structure of the chain, the considering of share transaction and its price may complicate the discussions and lose the main point. Conversely, this simplified model can roughly capture the mechanism of shareholding in the vertical related supply chain, which has also proved to be reasonable (e.g., Fu and Ma 2019). In the extension section, we consider the existence of share price and show similar conclusions to our main text. Second, it is also noteworthy that the platform is a follower in our model, like Amazon and Samsung Galaxy S (Kward et al., 2014), and it can also be regarded as a weaker retailer than the supplier (Shen et al. 2019) like Jinghai compared with Gree. Meanwhile, some enterprises like Lao Gan Ma (a chili products supplier) can provide unique or extremely competitive products to the market, although these are quite smaller than the large retailers like Taobao and JD.com, they can also take the high ground in supply chain competitions. Moreover, this structure can also be applied in the supply chain that the e-tailers like Tianxianpei and Haier's online distributers that are relatively weaker.

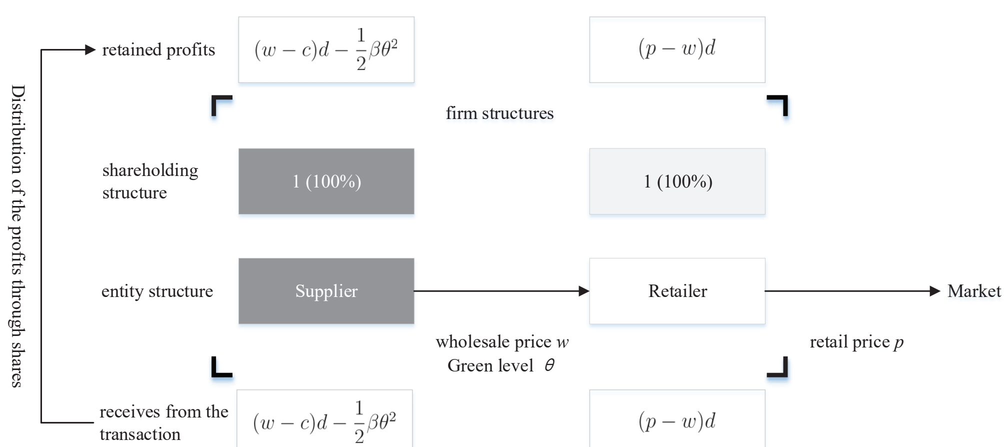


Figure 1: Supply chain without shareholding (traditional reselling scheme)

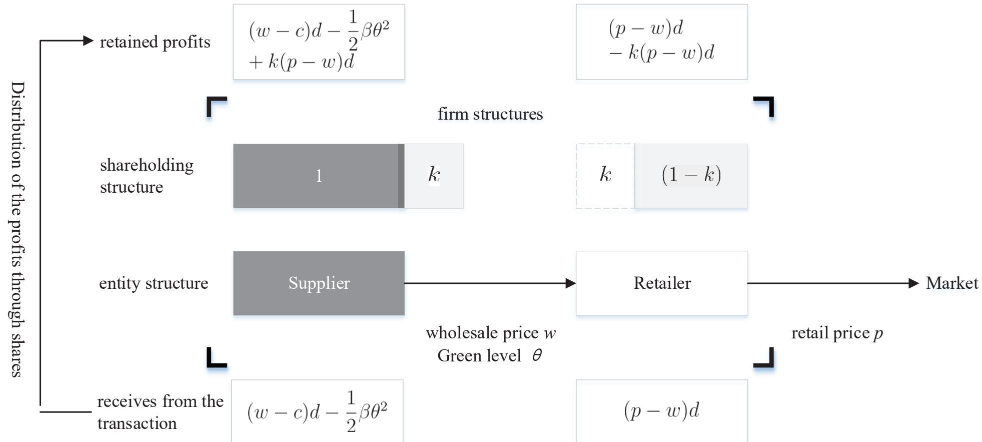


Figure 2: Supply chain with forward shareholding strategy

The sequence of events in the shareholding mode takes place as follows. Before the production and transaction periods, shareholding rates combining with shareholding strategy are settled. In the first stage, the supplier decides the green level for the product and the wholesale price to the platform retailer. Then in the second stage, the platform retailer determines its optimal retail price with respective to the wholesale price and green level. In the third stage, all products available are sold out without any retention. At last, the profits are distributed according to the shareholding rates that settled previously.

3.2. Reselling scheme (Case R*)*

Firstly, a reselling model as a benchmark is established in which both players are completely independent. In this situation, the products are purchased from the supplier and sold to the consumers by the platform, and none shareholding strategies are considered. For convenience, we use the superscript R (reselling) to denote this scheme. Therefore, the profits of the supplier and the platform retailer can be shown as

$$\pi_s^{\mathrm{R}} = (w-c)d - \frac{1}{2}\beta\theta^2 \tag{1}$$

and

$$\pi_r^{\mathrm{R}} = (p-w)d, \tag{2}$$

respectively. By solving this Stackelberg game, the optimal green level, wholesale price and retail price for the players can be obtained as follows.

$$\begin{cases} \theta^{\mathrm{R}*} = \dfrac{a(bc-A)}{a^2-4b\beta}, \\ w^{\mathrm{R}*} = \dfrac{(-2bc-2A)\beta + a^2c}{a^2-4b\beta}, \\ p^{\mathrm{R}*} = \dfrac{(-bc-3A)\beta + a^2c}{a^2-4b\beta}. \end{cases} \tag{3}$$

Substituting the above equations in the supplier and the platform retailer's objective functions (1) and (2) yields

$$\pi_s^{\mathrm{R}*} = \frac{\beta(A-bc)^2}{8b\beta - 2a^2} \tag{4}$$

and

$$\pi_r^{\mathrm{R}*} = \frac{b\beta^2(A-bc)^2}{(a^2-4b\beta)^2}. \tag{5}$$

3.3. Shareholding scheme (Case S*)*

Based on the traditional reselling scheme, shareholding scheme (S) containing three shareholding strategies is raised in this supply chain. Specifically, when $j = 0$ and $k > 0$, only the retailer's share is held by the supplier, the supply chain manifests as *forward shareholding*. Similarly, when $k = 0$ and $j > 0$, *backward shareholding* arises. When k and j are both positive, the relationship between the supplier and the retailer is *cross-shareholding*. Similar to subsection 3.1, the objective functions, optimal solutions and final profits of both players are proposed in turn under different of shareholding strategies.

*3.3.1. Forward shareholding strategy (*SF*)*

When $j = 0$ and $k > 0$, the supplier holds k parts of the platform retailer's share and the platform retailer holds $(1-k)$ parts of its own share. In this case, the supplier can build a closer relationship with the consumers while benefiting from their green sensitivity more directly. Let the superscript SF (shareholding scheme, forward strategy) represents this case and the supplier's objective function changes as

$$\pi_s^{\mathrm{SF}} = (w-c)d - \frac{1}{2}\beta\theta^2 + k(p-w)d, \tag{6}$$

and the platform retailer's profit falls to $(1-k)$ proportion as compared to the benchmark R as

$$\pi_r^{\mathrm{SF}} = (1-k)(p-w)d. \tag{7}$$

Following the same procedure as stated in the preceding section, the optimal green level and wholesale price for the supplier as well as the optimal retail price for the platform retailer can be obtained, such that

$$\begin{cases} \theta^{\mathrm{SF}*} = \dfrac{a(bc-A)}{a^2-2b(2-k)\beta}, \\ w^{\mathrm{SF}*} = \dfrac{a^2c - (2bc+2A(1-k))\,\beta}{a^2-2b(2-k)\beta}, \\ p^{\mathrm{SF}*} = \dfrac{a^2c + (A(2k-3)-bc)\,\beta}{a^2-2b(2-k)\beta}. \end{cases} \tag{8}$$

Utilizing the above optimal solutions to the objective functions of the supplier and platform retailer, their respective profits

$$\pi_s^{\mathrm{SF}*} = \frac{\beta(A-bc)^2}{4b(2-k)\beta - 2a^2}, \tag{9}$$

and

$$\pi_r^{\text{SF}*} = \frac{(A-bc)^2(1-k)b\beta^2}{\left(a^2-2b(2-k)\beta\right)^2}. \tag{10}$$

*3.3.2. Backward shareholding strategy (*SB*)*

Similarly, considering the case where $k = 0$ and $j > 0$. That is, the platform retailer holds j proportion of the supplier's shares and the supplier holds only its own shares. This means that backward shareholding (with the superscript SB) is addressed, and hence the profits of the two parties become

$$\pi_s^{\text{SB}} = (1-j)\left((w-c)d - \frac{1}{2}\beta\theta^2\right) \tag{11}$$

and

$$\pi_r^{\text{SB}} = (p-w)d + j\left((w-c)d - \frac{1}{2}\beta\theta^2\right). \tag{12}$$

Then we can get the optimal solutions as follows.

$$\begin{cases} \theta^{\text{SB}*} = \dfrac{a(A-bc)}{4b(1-j)\beta - a^2}, \\ w^{\text{SB}*} = \dfrac{a^2c + \left((4j-2)bc - 2A\right)\beta}{a^2 - 4b(1-j)\beta}, \\ p^{\text{SB}*} = \dfrac{a^2c + 3(j-1)\left(A + \frac{1}{3}bc\right)\beta}{a^2 - 4b(1-j)\beta}. \end{cases} \tag{13}$$

The corresponding profits of the supplier and the platform retailer are

$$\pi_s^{\text{SB}*} = \frac{(A-bc)^2(j-1)\beta}{2a^2 - 8\beta(1-j)b} \tag{14}$$

and

$$\pi_r^{\text{SB}*} = \frac{1}{2}\frac{(A-bc)^2\left(2\beta(j-1)^2b - a^2j\right)\beta}{\left(a^2-4b(1-j)\beta\right)^2}. \tag{15}$$

*3.3.3. Cross-shareholding strategy (*SC*)*

Combining the above two single-way shareholding strategies, the cross-shareholding strategy (with superscript SC) is introduced in this subsection. It possesses the characteristics of both forward shareholding and backward shareholding. With this strategy, the objective functions of the supplier and the platform retailer convert to

$$\pi_s^{\text{SC}} = (1-j)\left((w-c)d - \frac{1}{2}\beta\theta^2\right) + k(p-w)d \tag{16}$$

and

$$\pi_r^{\text{SC}} = (1-k)(p-w)d + j\left((w-c)d - \frac{1}{2}\beta\theta^2\right). \tag{17}$$

The optimal operating decisions for the players are

$$
\begin{cases}
\theta^{\mathrm{SC}*} = \dfrac{a(A-bc)}{2\beta(k-2)(j-1)b-a^2}, \\
w^{\mathrm{SC}*} = \dfrac{a^2c(j+k-1)-2(j-1)\left(bc(k-2)j+(k-1)(Ak-bc-A)\right)\beta}{(j+k-1)\left(a^2-2\beta(k-2)(j-1)b\right)}, \\
p^{\mathrm{SC}*} = \dfrac{a^2c-2(j-1)\left(-\dfrac{1}{2}bc+A\left(k-\dfrac{3}{2}\right)\right)\beta}{a^2-2\beta(k-2)(j-1)b}.
\end{cases} \tag{18}
$$

Thus, the profits of the supplier and the platform retailer can be obtained as

$$\pi_s^{\mathrm{SC}*} = \frac{(A-bc)^2(j-1)\beta}{2a^2-4\beta(k-2)(j-1)b} \tag{19}$$

and

$$\pi_r^{\mathrm{SC}*} = \frac{1}{2}\frac{(A-bc)^2\left(2\beta(j-1)^2(1-k)b-a^2j\right)\beta}{(a^2-2\beta(k-2)(j-1)b)^2}. \tag{20}$$

Based on the above calculation results, the optimal solutions under different shareholding strategies combining with the reselling benchmark can be investigated. Based upon these conclusions, the profits of both the players and the whole supply chain can be also examined.

4. Optimal Decisions of the players

On the basis of the results above, the optimal decisions of the supplier and the retailer are discussed in this section. Specifically, we focus on the impacts of shareholding rates upon the the optimal green level and wholesale price of the supplier and retail price of the retailer. That is, the monotonicity of both players' optimal decisions with respect to both shareholding rates are proposed at first. Followed by the corresponding monotonicity, the optimal solutions of both players under traditional reselling and all shareholdings strategies are also compared.

4.1. Supplier's optimal decisions

In this part, we first investigate the effects of shareholding rates on the supplier's optimal green level and then focus on its wholesale price decision.

4.1.1. Optimal green level

In this part, the monotonicity and relative size of the optimal green level under the benchmark and various shareholding strategies are discussed. In particular, Proposition 1 (i) shows the monotonicity of the green level to the corresponding shareholding rates, and Proposition 1 (ii) compares the optimal green levels under reselling and different shareholding strategies.

Proposition 1. *The impacts of shareholding rates k and j on the optimal green level and the corresponding comparisons are as follows.*

(i) $\dfrac{\partial\theta^{n*}}{\partial k} > 0$, $\dfrac{\partial\theta^{n'*}}{\partial j} > 0$; $\dfrac{\partial\theta^{n'*}}{\partial j} > \dfrac{\partial\theta^{n*}}{\partial k}$, *if* $j = m = k$ $(n \in \{\mathrm{SF},\ \mathrm{SC}\},\ n' \in \{\mathrm{SB},\ \mathrm{SC}\})$;

(ii) *if* $j < \dfrac{1}{4}$ *and* $k > 2j$, $\theta^{\mathrm{SC}*} > \theta^{\mathrm{SF}*} > \theta^{\mathrm{SB}*} > \theta^{\mathrm{R}*}$, *otherwise* $\theta^{\mathrm{SC}*} > \theta^{\mathrm{SB}*} > \theta^{\mathrm{SF}*} > \theta^{\mathrm{R}*}$.

Proposition 1 (i) shows that the increase of shareholding rates always leads to a higher green level of the supply chain, showing a similar role in greenness promotion like the cost/revenue sharing ratio (Ghosh and Shah, 2012; Giovanni, 2014). However, the mechanisms behind shareholdings are more complex than the simple sharing for cost/revenue. For the forward shareholding rate, since the supplier can acquire the benefits from green sensitive consumers more directly through holding share effects, it will increase the green level to boost the demand and raise the value of the retailer's share. For the backward shareholding rate, although the supplier incurs a loss under the impacts of holding share effects, the incentive effects from a more favorable decision of the retailer (i.e., lower retail price) employ the supplier a higher optimal green level to choose. These insights confirm the conclusions that forward shareholding contributes to upstream investment of related research (e.g., Letizia and Hendrikse 2016) and also extend them to the case of backward shareholding. Furthermore, combining the above results, interestingly, we observe that the green level is more sensitive to the backward shareholding rate when both rates are at the same level, which may be kind of counter intuitive. This observation shows the important role of indirect incentive of the shareholding in green investment decisions. On the whole, when a supply chain or its regulator wants to increase the greenness of the chain, increasing the shareholding rates, especially the backward one might be a feasible method.

By comparing the optimal green levels of all the strategies used as per Proposition 1 (ii), we can easily deduce that the green levels in all shareholding strategies are strictly higher than that achievable in the traditional reselling scheme, demonstrating an important effect of shareholdings in the investments. Generally speaking, a tighter relationship (i.e., higher k, higher j or both) between the two players will stimulate the supplier to invest more in the greenness of the products, leading to a higher green level overall and achieving the highest green level in the cross-shareholding strategy. Meanwhile, as the optimal green level is more sensitive to the backward shareholding rate rather than the forward rate, the forward shareholding's optimal green level will be smaller than the backward one unless it exceeds a certain threshold. It shows a higher efficiency of the backward shareholding than the forward one to the greenization of the supply chain.

4.1.2. Optimal wholesale price

Similar to the above analysis, the monotonicity and comparison of wholesale prices under different shareholding strategies are addressed.

Proposition 2. *The impacts of shareholding rates k and j on the optimal wholesale price and the corresponding comparisons are as follows.*

(i) $\frac{\partial w^{\mathrm{SF}*}}{\partial k} < 0$; $\frac{\partial w^{\mathrm{SB}*}}{\partial j} > 0$; $\frac{\partial w^{\mathrm{SC}*}}{\partial j} > 0$, *and if* $k < k_{w1} = 1 - \frac{2j\left(a^2 - 2b\beta(1-j)\right)}{a^2 - 2b\beta(1-j)^2}$, $\frac{\partial w^{\mathrm{SC}*}}{\partial k} < 0$, *otherwise* $\frac{\partial w^{\mathrm{SC}*}}{\partial k} > 0$;

(ii) (a) *when* $j < \frac{1}{2} - \frac{a^2}{4b\beta}$, *if* $k > k_{w2}$, $w^{\mathrm{SF}*} < w^{\mathrm{R}*} < w^{\mathrm{SC}*} < w^{\mathrm{SB}*}$; *otherwise* $w^{\mathrm{SF}*} < w^{\mathrm{SC}*} < w^{\mathrm{R}*} < w^{\mathrm{SB}*}$;

(b) *when* $j > \frac{1}{2} - \frac{a^2}{4b\beta}$*, if* $k > k_{w3}$*,* $w^{\mathrm{SF}*} < w^{\mathrm{R}*} < w^{\mathrm{SC}*} < w^{\mathrm{SB}*}$*; otherwise* $w^{\mathrm{SF}*} < w^{\mathrm{R}*} < w^{\mathrm{SB}*} < w^{\mathrm{SC}*}$*.*

Characteristics (i) in Proposition 2 demonstrates that the wholesale price is always decreasing in response to the forward shareholding rate and increasing to the backward rate under single-way shareholdings. The reasons behind these are as follows. Under forward shareholding strategy, as the supplier gains more profits from per unit of product through holding the retailer's shares, it has incentive to decrease the wholesale price to encourage a higher demand and benefit more from the higher holding share effects. Under backward shareholding strategy, the supplier suffers from the holding share of the retailer and inclines to raise the wholesale price to obtain a higher marginal. Under cross-shareholding strategy, the supplier's optimal wholesale price is also increased to the backward shareholding rate, which denotes that the holding share effects under backward shareholding always dominates under cross-shareholding strategy, leading a wholesale price increasing to the backward shareholding rate. However, the supplier's wholesale price shows a non-monotonously to the forward shareholding rate under cross shareholding. Specifically, when the forward shareholding rate is at a low level (i.e., k is lower than k_{w1}), the holding share effects are huge and the supplier acts the same as the single forward shareholding. Nevertheless, when forward shareholding rate continues to increase, lower the wholesale price becomes less efficient and therefore cannot benefit the supplier. This is because the supplier loss a partial of shares under cross-shareholding and the incentive effects are not so strong under this circumstance. Besides, Proposition 2 (ii) compares the optimal wholesale price under traditional reselling and that when using different shareholding strategies. It is observed that forward shareholding leads to a lower price than the traditional reselling scheme, whilst using backward strategy leads to higher wholesale price than this benchmark. Meanwhile, because of the optimal wholesale price's non-monotonicity to the shareholding rates while using cross-shareholding strategy, the wholesale price in that case may be lower or higher than the benchmark with the backward shareholding strategy depending on the structure of shareholding. If the backward shareholding rate is small, the wholesale price in cross-shareholding strategy will be lower than that achievable using backward shareholding alone and may even be smaller than that attainable in the benchmark case R if the forward shareholding rate is also small. If the backward shareholding rate is large, a larger wholesale price results in the cross-shareholding strategy as compared to the benchmark case. Furthermore, if the forward shareholding rate is also large, the wholesale price in cross-shareholding will become smaller than that obtainable with the backward one and become larger when the forward shareholding rate is small. The reason behind this is that the wholesale price in cross-shareholding may be increased in response to the forward shareholding rate in certain cases, leading the wholesale price to become the largest. That is, if the supplier losses a large part of share and can obtain much from the retailer's share, it will raise its wholesale price to a greatly high level to maintain the profits from the wholesale progress rather than lower the price to acquire higher incentive effects.

4.2. Retailer's optimal retail price

This subsection discusses issues related to the retail price decision from the platform retailer's viewpoint. Through an analysis of the results given in the preceding section, Proposition 3 can be obtained as follows.

Proposition 3. *The effects of shareholding rates k and j on the optimal retail price are as follows.*

(i) $\frac{\partial p^{\mathrm{SF}*}}{\partial k} < 0$, $\frac{\partial p^{\mathrm{SB}*}}{\partial j} > 0$, $\frac{\partial p^{\mathrm{SC}*}}{\partial j} > 0$, $\frac{\partial p^{\mathrm{SC}*}}{\partial k} < 0$*;*

(ii) *when* $k > \frac{3a^2 j}{2(j-1)(a^2 - b\beta)}$, $p^{\mathrm{SF}*} < p^{\mathrm{SC}*} < p^{\mathrm{R}*} < p^{\mathrm{SB}*}$*; otherwise,* $p^{\mathrm{SF}*} < p^{\mathrm{R}*} < p^{\mathrm{SC}*} < p^{\mathrm{SB}*}$*.*

Proposition 3 (i) shows that the forward shareholding rate is of negative impact upon the retail price while the backward shareholding rate is of positive impact regardless of the shareholding strategy. It is because the retailer should follow the supplier's decisions in a large extend as the former is in an inferior position in this supply chain. For example, as the supplier decreases the wholesale price under a higher forward shareholding rate, the retailer will follow it and also decrease the retail price. However, unlike the wholesale price may increase to the forward shareholding rate in certain circumstance under cross-shareholding strategy, the retail price is strictly negative to the forward shareholding rate under cross-shareholding strategy. In Proposition 3 (ii), we find that when comparing the optimal retail prices of the platform retailer with the use of different strategies, the retail price under the forward shareholding strategy is always smaller than that of the benchmark and always larger than that under the backward shareholding strategy. For the optimal retail price when using the cross-shareholding strategy, the retail price can be either higher or lower than the one attainable by the benchmark case, depending on what level of both shareholding rates are adopted. On the whole, if such a green supply chain or the government inclines to lower (or higher) the final price, increasing the forward (or backward) shareholding rate might be an available way.

Interestingly, combining Propositions 2 and 3, the wholesale price and retail price usually showed changes in the same direction. That is, the supplier's pricing strategy can be transferred by the retailer in most cases, indicating that the retailer can compensate its cost by increasing the price to the ultimate consumer. However, owing to the non-synchronousness of both the wholesale and retail prices, a higher wholesale price does not necessarily imply a higher retail price. In particular, when the forward shareholding rate is sufficiently large (e.g., $k > k_{w1}$), the wholesale price and retail price will change in different directions $\left(\text{i.e., } \frac{\partial w^{\mathrm{SC}*}}{\partial k} > 0 \text{ while } \frac{\partial p^{\mathrm{SC}*}}{\partial k} < 0\right)$. This is because the platform retailer may sacrifice its marginal profits to stabilize the demand of the market given such a situation, through which the supplier may further shrink the margin of the platform retailer. These propositions also illustrate the superiority of the supplier in pricing over the platform retailer owing to the leadership in the Stackelberg game and through which the supplier can adjust its wholesale price more flexibly. In the practice, one supplier can increase the forward shareholding rate of its downstream retailer to further shirink the latter's margin if both firms are under cross-shareholding strategy. Conversely, the retailer should prevent its supplier's extremely high forward shareholding to maintain its unit profit. Furthermore, the combination of these propositions signifies that the cross-shareholding strategy is not a mere combination of the forward and backward shareholdings. When both forward and backward shareholdings exist, the pricing behaviors of the supplier and the platform retailer will interact with each other and may finally alter the relationships between shareholding rates and the profits of the players.

5. Optimal Shareholding Strategies for Both Players under Shareholding Scheme

This section discusses the performances of both players under different shareholding strategies. Specifically, the monotonicity of their profits is shown first, and the strategy selections of the participants are discussed in the later. Nnumerical examples are also provided to deliver our results more clearly. In the numerical examples, we let $A = 1$, $a = 0.2$, $b = 0.6$, $\beta = 0.15$, $c = 0.01$.

5.1. Impacts of shareholding rates on the players' profits

The participants' performances under various shareholding strategies are analyzed in this subsection and the following characteristics have been reached.

Proposition 4. *The impacts of shareholding rates k and j on the profits of supplier and platform retailer are as follows.*

(i) (a) $\frac{\partial \pi_s^{\mathrm{SF}*}}{\partial k} > 0$; (b) $\frac{\partial \pi_s^{\mathrm{SB}*}}{\partial j} > 0$; (c) $\frac{\partial \pi_s^{\mathrm{SC}*}}{\partial j} > 0$, $\frac{\partial \pi_s^{\mathrm{SC}*}}{\partial k} > 0$;

(ii) (a) *if* $k \in \left(0,\ \frac{a^2}{2b\beta}\right)$, $\frac{\partial \pi_r^{\mathrm{SF}*}}{\partial k} > 0$, *otherwise* $\frac{\partial \pi_r^{\mathrm{SF}*}}{\partial k} < 0$;

(b) *if* $j \in \left(0,\ \frac{a^2}{8b\beta}\right)$, $\frac{\partial \pi_r^{\mathrm{SB}*}}{\partial j} > 0$, *otherwise* $\frac{\partial \pi_r^{\mathrm{SB}*}}{\partial j} < 0$;

(c) *if* $j < \frac{1}{3}$ *and* $k \in \left(0,\ \frac{a^2(1-3j)}{2(1-j)^2 b\beta}\right)$, $\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial k} > 0$, *otherwise* $\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial k} < 0$; *if* $k < \frac{a^2}{2b\beta}$ *and* $j \in \left(0,\ \frac{2b\beta k - a^2}{2(3k-4)b\beta}\right)$, $\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial j} > 0$, *otherwise* $\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial j} < 0$.

Through item (i) of Proposition 4, it can be observed that the supplier's profits always increase in response to both shareholding rates under any type of shareholding strategy. This may due to its status as the leader in the Stackelberg game: Through adjusting green level and wholesale price, the supplier can always benefit from a tighter relationship of the chain. When the forward shareholding rate becomes higher, the supplier can gain more profits through taking part in the retailing progress (holding share effects). Meanwhile, this benefit allows the supplier to invest more in the greenness of the product, which will also favor the supplier's total profits. Both of these will cover the loss from the retailer negative incentive effects and the supplier can always improve its profit through rasing the forward shareholding rate. When the backward shareholding rate becomes higher, although the supplier may suffer a loss in its shares, it can acquire more profits from a higher wholesale price and a much higher green level. In other words, the incentive effects from the retailer are always higher than the loss of its own shares, leading to a increasing of the supplier's profits to this shareholding rate.

For the platform retailer discussed in Proposition 4 (ii), however, the effects of shareholding rates on its profits are not monotonic. When using a single-way shareholding strategy, the platform retailer may gain more profits if the shareholding rate is low but may suffer a loss if the rate becomes too high. In particular, if $k < \frac{a^2}{2b\beta}$ $\left(j < \frac{a^2}{8b\beta}\right)$, the profits of the platform retailer will increase with the forward (backward) shareholding rate, otherwise it will decrease with it. For the forward shareholding, the incentive effects dominate when the shareholding rate is low and holding share effects become more distinct when the rate is high. For the backward shareholding, similarly, the negative incentive effects from backward shareholding may exceed the positive holding share effects when this rate becomes sufficiently high. Combining both single-way shareholdings, it can be noted that the threshold under forward shareholding is larger than the backward one, implying that the platform retailer can tolerate a higher rate for the forward shareholding than the backward one. In other words, despite there are no holding costs, the platform retailer may have no incentives to increase the shareholding rate of its supplier. This supports the phenomenon that the leader in a supply chain often holds a larger part of share than its follower in the practice. In adopting the cross-shareholding strategy, the issue of profit monotonicity is more complex because the interaction between the two shareholding rates. Specifically, for the forward shareholding rate k's impacts, when the backward shareholding rate is sufficiently small $\left(\text{i.e., } k > \frac{1}{3}\right)$, the retailer's profits remain the similar character to the forward shareholding strategy. However, when the backward shareholding rate is large, the retailer's profits will always decline to the forward shareholding rate, which are consistent with (Chen et al., 2016). This inidicates that, when a large part share of the supplier is held by the retailer, the supplier may more focus on its own remaining shares. Therefore, on the view of the retailer, the incentive effects from a higher forward shareholding rate is quite small and cannot cover its loss in the profits from its own shares. For the impact of backward shareholding rate, we note the following observations. If the forward shareholding rate exceeds a certain level, the retailer's profits will always decrease to the backward shareholding rate, otherwise the profits will increase to it first and then decrease. The reason behind it is that, when applying a sufficiently high forward shareholding rate, incentive effects have been extracted from it in a large extent. Therefore, holding share effects from increasing the backward shareholding rate will be always smaller than the grown negative incentive effects. It is also noteworthy that, the retailer's tolerances to both shareholding rates becomes lower than those in the single-way shareholdings.

5.2. Strategy preferences of the players

Following the discussion in 5.1, we can obtain the players' preferences of shareholding strategies as follows.

Proposition 5. *Supplier's strategy preferences under different levels of shareholding rates k and j satisfies that*
if $k < \dfrac{a^2 j}{2(1-j)b\beta}$, $\pi_s^{\mathrm{R}*} < \pi_s^{\mathrm{SF}*} < \pi_s^{\mathrm{SB}*} < \pi_s^{\mathrm{SC}*}$*; otherwise* $\pi_s^{\mathrm{R}*} < \pi_s^{\mathrm{SB}*} < \pi_s^{\mathrm{SF}*} < \pi_s^{\mathrm{SC}*}$.

Proposition 5 compares the supplier's profits in using different shareholding strategies. Not surprisingly, while the profit of using a single-way shareholding strategy is always higher than that attainable in the benchmark scenario, the employment of cross-shareholding strategy will provide the supplier the most profit under the same level of single-way shareholding rates. Additionally, in comparing the two single-way shareholding strategies, if k is sufficiently high compared with the backward shareholding rate j, the profit of the supplier in the forward shareholding strategy will be higher than that in the backward one, otherwise, the profit of under backward shareholding will be higher. These above indicate the supplier's preference for high shareholding rates under any shareholding strategy, leading cross-shareholding strategy to be the supplier's optimal strategy.

Proposition 6. *Platform retailer's strategy preference under different levels of shareholding rates k and j satisfies that*
(i) *if* $k \in (0,\ \dfrac{4a^2b\beta - a^4}{4b^2\beta^2})$, $\pi_r^{\mathrm{SF}*} > \pi_r^{\mathrm{R}*}$*, otherwise* $\pi_r^{\mathrm{SF}*} < \pi_r^{\mathrm{R}*}$*; if* $j \in (0,\ \dfrac{a^4 - 4a^2b\beta}{2a^2b\beta - 16b^2\beta^2})$, $\pi_r^{\mathrm{SB}*} > \pi_r^{\mathrm{R}*}$,
otherwise $\pi_r^{\mathrm{SB}*} < \pi_r^{\mathrm{R}*}$*; if* $k \in (k_3,\ k_4)$, $\pi_r^{\mathrm{SC}*} > \pi_r^{\mathrm{R}*}$*, otherwise* $\pi_r^{\mathrm{SC}*} < \pi_r^{\mathrm{R}*}$*;*
(ii) *if* $k \in (k_1,\ k_2)$, $\pi_r^{\mathrm{SB}*} < \pi_r^{\mathrm{SF}*}$*, otherwise* $\pi_r^{\mathrm{SB}*} > \pi_r^{\mathrm{SF}*}$*;*
(iii) *if* $k > k_5$, $\pi_r^{\mathrm{SC}*} < \pi_r^{\mathrm{SF}*}$*, otherwise* $\pi_r^{\mathrm{SC}*} > \pi_r^{\mathrm{SF}*}$*; if* $k > k_6$, $\pi_r^{\mathrm{SC}*} < \pi_r^{\mathrm{SB}*}$*, otherwise* $\pi_r^{\mathrm{SC}*} > \pi_r^{\mathrm{SB}*}$

Based on Proposition 6, Corollary 1 is provided as below.

Corollary 1. *Retailer's choice of the cross-shareholding strategy.*
If $j < \dfrac{t(t-4)}{2t-16}$*, and* $k \in (k_3,\ \min\{k_4,\ k_5,\ k_7\})$, $\pi_r^{\mathrm{SC}*} > \max\{\pi_r^{\mathrm{SF}*},\ \pi_r^{\mathrm{SB}*},\ \pi_r^{\mathrm{R}*}\}$*, notice that* $t = \dfrac{a^2}{b\beta}$.

Unlike the supplier's preference in shareholding strategies as shown in Proposition 5, the platform retailer cannot always benefit from the shareholding strategies and will only acquire more profit if the shareholding rate k or j is low in the single-way shareholding strategies as displayed in Proposition 4. Therefore, the profits under single-way shareholding strategies will be higher than under traditional reselling only when the shareholding rates are lower than certain thresholds as displayed in Proposition 6 (i). Regarding the cross-shareholding strategy, the platform retailer will benefit from the shareholding if both the shareholding rates k and j are in a specific interval. While the comparison of the profits under all the schemes or strategies are too complex and the supplier always prefers the cross-shareholding strategy, we only focus on the retailer's preference to the cross-shareholding strategy and Corollary 1 goes by. We observe that, when the backward shareholding rate is sufficiently low and the forward shareholding rate is under certain level, cross-shareholding will be the retailer's optimal strategy.

5.3. Pareto-region(s) of the supplier and the retailer

On the base of the above subsections, we then discuss the existence of Pareto-region(s) of all the supply members in Corollary 2.

Corollary 2. *Pareto-region exists under certain level of shareholding rates.*

Observing from Propositions 5, 6 and Corollary 1, both the supplier and the retailer inclines to cross-shareholding under certain level of shareholding rates k and j.

Combining Propositions 5 and 6, interestingly, both parties in the chain will choose cross-shareholding as their optimal strategy when the shareholding structure is in certain intervals. However, unlike the existence literature's conclusion that buyer enjoys cross-shareholding more (Güth et al., 2007), our discussion shows that the seller is more delightful to cross-shareholding. This difference is mainly because the different market position settings of the supplier and retailer. To illustrate these results more clearly, assume that the two shareholding rates are not fixed while other parameters remain unchanged. While the supplier always prefers cross-shareholding, we only provide the retailer's preference to the shareholding strategies as depicted in Fig. 3.

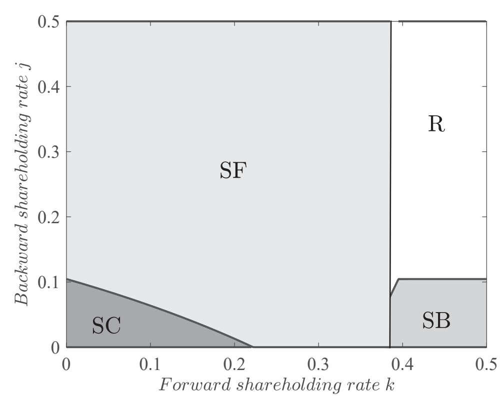


Figure 3: Shareholding strategy preference of the platform retailer under different level of shareholding rates k and j

Figure 3 shows the strategy preference for the platform retailer varies in a number of ways. First, when both shareholding rates are large, particularly in the area R, the platform retailer will end up with a loss in any case of shareholding, thereby tending to remain totally independent. Second, the forward shareholding (SF) will be the optimal strategy for the platform retailer in most cases. The reason is that although experiencing losses in the holding share effects (be held), the platform retailer will obtain a more favorable wholesale price and a higher green level. That is, the incentive effects plays a vital important role in the shareholding strategy choice of the retailer. Moreover, if the forward shareholding rate is extremely large and the backward one is small (area SB), it will choose the backward shareholding strategy. This is because when the forward shareholding rate is extremely high, the supplier will further extract the margin of the retailer and the incentive effects become dominated. Under this situation, a much lower backward shareholding rate can provide more profits for the retailer. Last but not least, the platform retailer will choose cross-shareholding in the area SC in which both rates are low. This is because the platform retailer will benefit from both shareholding rates when they are sufficiently low as shown in the previous propositions. Considering the supplier's optimal strategy, cross-shareholding may result in a win-win situation at low cross-shareholding levels, which shows a similar outcome with (Fu and Ma, 2019) under different settings, and also explain the fact that the shareholding rates of cross-shareholding are generally not very high in practice like the Japanese market.

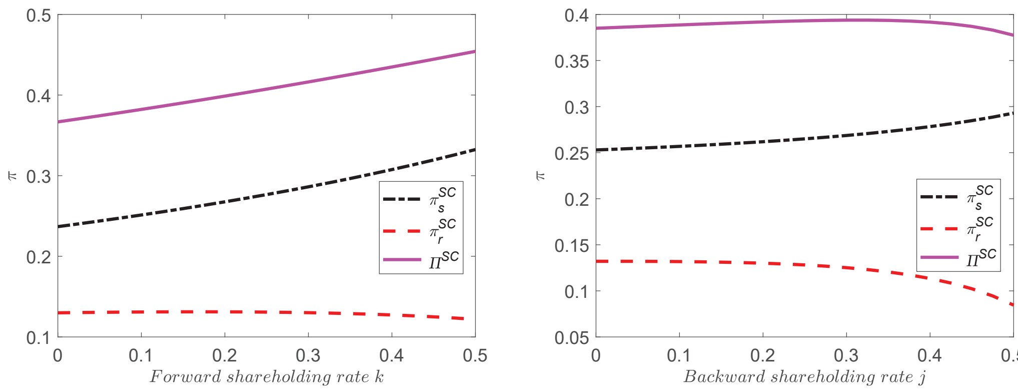
6. Shareholding Strategy from the View of Whole Supply Chain

In this section, we investigate the shareholding strategies from the view of the whole supply chain. We first advance the impacts of shareholding rates on the chain's profit and then discuss how to improve the whole supply chain's performance without injuring any participants. In other words, Pareto-improvement's achievement is analyzed under the shareholding model we proposed. While the chain achieves cross-shareholding as discussed in the former sections, we only focus on the characters of the chain's profit under cross-shareholding in this part.

Proposition 7. *The effects of shareholding rates on the entire supply chain's profit are as follows.*

$\frac{\partial \Pi^{SC}}{\partial k} > 0$*; when* $j < \frac{1-k}{3-2k}$, $\frac{\partial \Pi^{SC}}{\partial j} > 0$*, otherwise* $\frac{\partial \Pi^{SC}}{\partial j} < 0$.

It's noticed that the supply chain's profit is not monotonic to both shareholding rates as indicated in the above proposition. First, a higher forward shareholding rate always leads to a better performance of the chain, which reaches a good agreement with the existence research (Gaigné et al., 2018; Fu et al., 2018). Second, we also compensate these works by showing that a higher backward shareholding rate will decrease the chain's profit when it exceeds a certain level. Besides, compared with the threshold in Proposition 4 (ii), it is known that $\frac{1-k}{3-2k} > \frac{2b\beta k - a^2}{2(3k-4)b\beta}$. That is, when the backward shareholding rate is at a certain level $\left(\text{i.e., } \frac{2b\beta k - a^2}{2(3k-4)b\beta} < j < \frac{1-k}{3-2k}\right)$, the entire supply chain's profit will increase to the backward shareholding rate at the cost of hurting the retailer. If the backward shareholding rate continues to rise $\left(\text{when } j > \frac{1-k}{3-2k}\right)$, the retailer's loss will surpass the supplier's earnings and eventually damage the entire supply chain. Figure 4 illustrates these results in more detail (it assumes $j = 0.15$ or $k = 0.15$, respectively to show the other shareholding rate's influence on the profit).



(a) Supplier, platform retailer and chain's profit under different forward shareholding rates k

(b) Supplier, platform retailer and chain's profits under different backward shareholding rates j

Figure 4: Supplier, platform retailer and chain's profits under different forward and backward shareholding rates k and j

Facing the inequality the turning points of the retailer and whole supply chain in Propositions 4 (ii) nad 7, two distinct ways to coordinate the supply chain can be proposed as follows.

Corollary 3. *Pareto improvement can be achieved if the supplier can pay a premium for the forward shareholding.*

In Corollary 3, it shows if the supplier can pay for the retailer's share under cross-shareholding, Pareto-improvement may be achieved under certain circumstance. Although raising the forward shareholding rate is harmful to the retailer, the whole supply chain will always benefit from a higher forward shareholding rate, which means that the supplier's gain from raising forward shareholding rate is always higher than the retailer's loss. Therefore, if the supplier can sacrifice part of its profits, for example, in the sense of paying for the retailer's shares, the forward shareholding rate of the chain can be improved and both the supplier and the retailer's performances can be (weakly) improved.

Corollary 4. *Pareto improvement can be achieved if the supplier can compensate the retailer's backward shareholding under certain circumstances.*

Likewise, observing Corollary 4, when the supplier can compensate (e.g., charge a negative price) for the retailer's backward shareholding in some occasions, both the players' performances can be improved and Pareto improvement goes by. Specifically, under this circumstance, the retailer will gain more profits from holding supplier's share and thus accept higher backward shareholding rates. Although the supplier loss more in compensating the retailer's backward shareholding, a higher backward shareholding rate's benefits can cover this loss and bring more profits through setting a proper compensation level. These two methods can be witnessed as similar equity incentive methods compared with the ones of Haier and Luzhou Laojiao, that is, providing shares for their retailers that even significantly lower than their corresponding shares market values.

7. Extensions

In this part, we first investigate the shareholding scheme's optional at the existence of traditionally believed most competitive scheme—agency selling in the platform area. Secondly, the zero share price assumption is relaxed in the second subsection. At last, we extend the green investment cost to another form to check the robustness of our conclusions.

7.1. Compare with shareholding and agency selling

7.1.1. Agency selling scheme

In this subextension, we compare the shareholding scheme with the agency selling scheme and discuss whether the shareholding scheme can be achieved facing considered competitive agency selling, the comparison of whole chain's performances under both schemes is also proved. Following the classic agency selling model, we use x ($x \in (0,\ 1)$) to denote the sharing ratio that the platform retailer obtained (Shen et al., 2019; Zhang et al., 2019; Yu et al., 2020). That is, x proportion is distributed to the platform retailer and the $(1-x)$ part left is kept by the supplier for per profit gains from the agency selling. Then, the profit of the supplier is

$$\pi_S^A = (1-x)pd - cd - \frac{1}{2}\beta\theta^2, \tag{21}$$

while the platform retailer gains

$$\pi_R^A = xpd. \tag{22}$$

Under this situation, the supplier will set the optimal green level and retail price (Yu et al., 2020) to maximize its profit as

$$\theta^{\mathrm{A}*} = \frac{(-Ax - bc + A)a}{a^2x - a^2 + 2b\beta} \tag{23}$$

and

$$p^{\mathrm{A}*} = \frac{-a^2cx + A\beta x + a^2c - b\beta c - A\beta}{a^2x^2 - 2a^2x + 2b\beta x + a^2 - 2b\beta}. \tag{24}$$

Through agency selling, the wholesale progress is omitted and the free riding effect is ignored. However, the supplier (platform retailer) may not choose this scheme when the sharing ratio x is quite high (low) and thereby it may gain little profit from this scheme. To save the space, we focus on the performances of the players and the whole supply chain in this part. The optimal decisions compared with the traditional reselling scheme and the cross-shareholding strategy in the shareholding scheme are consigned to Appendix D.

7.1.2. Players' preferences between shareholding and agency selling

We first focus on the impacts of agency selling sharing ratio on both players' profits and then discuss their preferences to the cross-shareholding or agency selling.

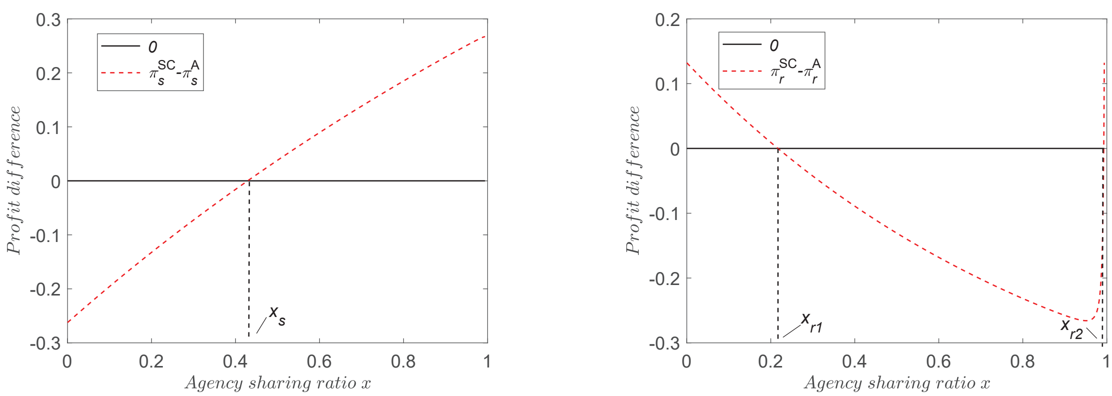
Proposition 8. *The impacts of sharing ratio on the supplier and retailer's profits are as follows. To ensure the positive of the optimal solutions, we have* $x < \dfrac{A-bc}{A}$ *in the following discussions.*

(i) $\dfrac{\partial \pi_s^{\mathrm{A}*}}{\partial x} < 0$;

(ii) *if* $x \in (0,\ x_3)$, $\dfrac{\partial \pi_r^{\mathrm{A}*}}{\partial x} > 0$, *otherwise* $\dfrac{\partial \pi_r^{\mathrm{A}*}}{\partial x} < 0$.

Proposition 8 shows that, while the supplier's profit is always decreasing to the agency sharing ratio, the retailer may not benefit from a higher level of it. For the supplier, the raising of sharing ratio always leads to a lower green level and smaller partial of revenue, leading to a decrease of the supplier's profits at all the time. However, although a higher sharing ratio seems to be beneficial to the retailer, increasing the sharing ratio cannot always improve its profits, which is owing to the concave retailer price. Specifically, when the sharing ratio is extremely large, the supplier should cover its cost in production and greenness investment through increasing the retail price, which will harm the sharing benefits of the retailer. While only cross-shareholding is optimal for both the supplier and the platform retailer, we further discuss the two participants' preferences to agency selling scheme and cross-shareholding and Proposition 9 can be obtained as follows.

Proposition 9. *Supplier and retailer's preferences to cross-shareholding and agency selling.*
(i) *if* $x < x_s$, *the supplier prefers agency selling, otherwise it inclines to choose the (cross) shareholding strategy;*
(ii) *if* $x \in (x_{r1},\ x_{r2})$, *agency selling is more profitable to the retailer, otherwise the shareholding will be better.*



(a) Supplier's scheme preference

(b) Platform retailer's scheme preference

Figure 5: Scheme preferences of supplier and platform retailer under different level of agency selling sharing ratio x and forward shareholding rate k

Proposition 9 and Fig. 5(a) indicates that, as the supplier strictly benefits to a lower sharing ratio x, it prefers agency selling when the sharing ratio is in a low level and favors cross-shareholding when the agency sharing ratio is high. For the platform retailer, however, its scheme choice can be more complex as displayed in Proposition 9 (ii) and Fig. 5(b). There exists three regions that the platform retailer will choose shareholding strategies. First, when the agency selling rate is extremely low, the platform retailer's profit in agency selling scheme will be less than the shareholding strategies and induce the platform retailer to choose the latter scheme. Secondly, with the raising of sharing ratio x, the agency selling scheme becomes more attractive to the retailer and will induce the retailer to select this scheme. Moreover, when the sharing ratio continues to grow, the supplier will adjust its optimal decisions and lower the marginal of unit products to increase the demand, leading to the decreasing of the retailer's profit under agency selling and thereby the shareholding strategies become more attractive to the platform retailer.

Based on the analysis above, we notice that the players will simultaneously select agency selling scheme when the sharing ratio is at a relatively low level (i.e., $x_{r1} < x < x_s$). Besides, both parties prefer shareholding strategy when the sharing ratio is extremely large (i.e., $x > \max\{x_s,\ x_{r2}\}$), which provides a new mechanism for the supplier and the platform retailer to corperate each other if the agency scheme becomes unavailable. On the whole, shareholding can also be achieved facing the alternative scheme of agency selling.

7.1.3. Scheme selection from the view of whole supply chain

In this part, from the viewpoint of the entire supply chain, we investigate the agency sharing ratio and shareholding rates' impacts on the chain's profit. The dominance relation between the agency selling scheme and cross-shareholding strategy is also revealed.

Proposition 10. *The impacts of agency sharing ratio on the supply chain.*
(i) $\dfrac{\partial \pi_c^{A*}}{\partial x} < 0$;
(ii) *if* $x < x_c$, $\pi_c^{A*} > \pi_c^{SC*}$, *otherwise* $\pi_c^{A*} < \pi_c^{SC*}$.

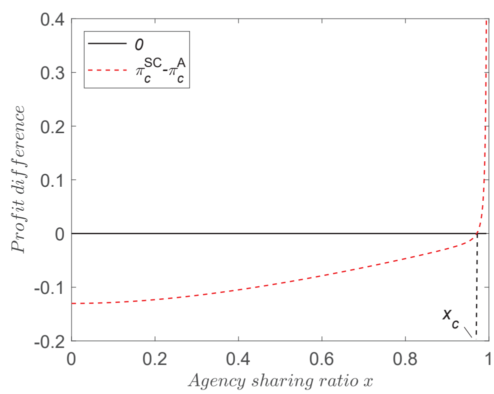


Figure 6: Chain's efficiency gap between agency selling and cross-shareholding under different sharing ratio x

Combining Figures 5 and 6, Observation 1 can be obtained as follows.

Observation 1. *Notice that* $x_c < x_{r2}$, *the supplier can compensate the retailer in certain forms to achieve the cross-shareholding in a larger area. This observation also echoes the conclusion in subsection 6.2.*

7.2. Endogenous share prices

In this subsection, we consider the shareholding strategies with the existence of share price. We assume that both of the firms' shares are priced based on their profits under benchmark scheme (traditional reselling). That is, the value of one firm is denoted by its total profit before all the shareholdings and we have $\pi_s^{R*} = \frac{(A-bc)^2\beta}{8b\beta-2a^2}$ and $\pi_r^{R*} = \frac{(A-bc)^2 b\beta^2}{(4b\beta-a^2)^2}$. The sequence of the events is as follows: In the first stage, the supplier/retailer decides what proportion of share to buy/sell to the another. After that, the decision progresses remain unchanged to that in subsection 3.1.

On account of the above description, the objective functions of both firms under forward shareholding strategy are

$$\pi_s^{SF'*} = (w-c)d - \frac{1}{2}\beta\theta^2 - k\frac{(A-bc)^2 b\beta^2}{(4b\beta-a^2)^2} + k(p-w)d \tag{25}$$

and

$$\pi_r^{SF'*} = (1-k)(p-w)d + k\frac{(A-bc)^2 b\beta^2}{(4b\beta-a^2)^2}. \tag{26}$$

Solving equations (25) and (26), we observe that, although the sharing prices have no impacts on the variable decisions, it will influence the strategy preference of the supplier/retailer for their impacts on the players' profits. Similarly, both players' profits under backward shareholding strategy and cross-shareholding strategy can also be obtained. Through the analyzing of their profits under different shareholding strategies, Proposition 11 is given below.

Proposition 11. *Under the existence of share price, the achievable shareholding rates under all the shareholding strategies can be described as follows.*
(i) *The forward shareholding rate can reach to its upper limit, i.e.,* $k = \frac{1}{2}$*;*
(ii) *The backward shareholding rate remains zero, i.e.,* $j = 0$*.*

When considering share price, it proves that both the supplier and the retailer always benefit from a higher forward shareholding rate in the domain of (0, 1/2). Thereby, both the players are delighted to raise the forward shareholding rate to its upper bound. The reasons behind this include two aspects: On the one hand, for the supplier, although incurs a loss in the shareholding transaction stage, the holding share effects becomes lower but still be positive owing to the cheap share price of the retailer. On the other hand, as we discussed in Section 6, the retailer's endurance to the forward shareholding rate is limited and its profit is concave to this rate. However, with the profits from the share transaction before the retailing, the retailer's maximum profit (profit function's vertices) is achieved at the limitation of shareholding rates (1/2). That is, the incentive effects and share transaction profit are always higher than the loss of its own shares. Under this scenario, a higher forward shareholding rate can always favors the retailer.

For Proposition 11 (ii), when considering share price, the retailer never benefits from the backward shareholding rate. This is because the retailer has to pay for its supplier's shares.

Thus lead to the optimal backward shareholding rate of the retailer gets smaller until to zero (the concave profit function of the retailer moves left). Then the backward shareholding cannot be applied by the retailer even it always benefit the supplier. Then it comes to the problem that, does it exist a method to achieve a positive backward shareholding rate? Inspired by the equity incentive in the reality, we propose a discount share price for the backward shareholding strategy.

While the backward shareholding can enhance the whole supply chain's performance and also benefits the supplier itself, the supplier has an incentive to encourage the downstream retailer to hold its shares. However, facing the expensive share price, the retailer will not adopt any backward shareholding project at all. Then the supplier could lower its share price to make the backward shareholding to be accepted by the retailer, and set its share price at $\frac{\delta(A-bc)^2 b\beta^2}{(4b\beta-a^2)^2}$ $(0<\delta<1)$, in which δ denotes the discount share price for the retailer as a financial incentive.

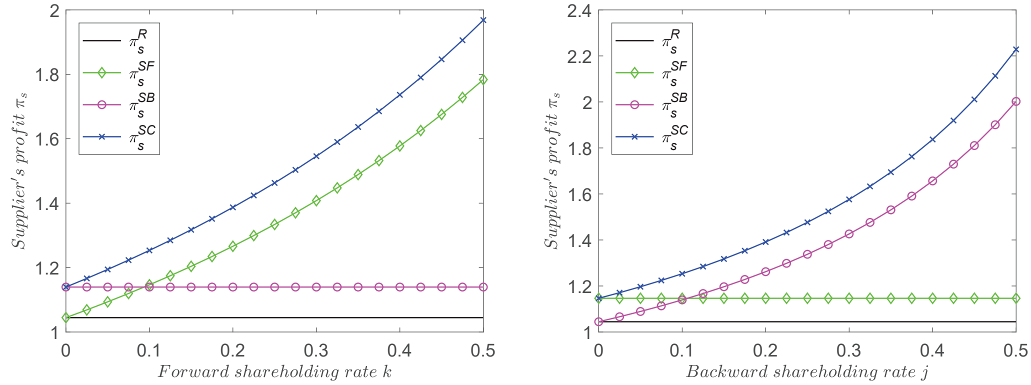
Proposition 12. *Cross-shareholding strategy remains optimal under certain level of shareholding rates for both players when the share price discount exists and in certain level, i.e.,* $\delta<\delta'=8\frac{a^2}{b\beta}(4-\frac{a^2}{b\beta})(-\frac{a^2}{8b\beta}+j)/(4j+\frac{a^2}{b\beta}-4)^3$.

Proposition 12 notices that when δ is sufficiently small, a proper backward shareholding rate can be accepted by the retailer. Thereby, cross-shareholding can be achieved combining the positive forward shareholding rate, which echoes our main conclusions. Our findings provides a novel proposal in this situation with a powerful supplier and upstream green investment: the powerful supplier can provide a share discount for its retailer to benefit both players of the chain.

7.3. Extension of the green investment cost

In this subsection, we check the robustness of our paper in changing the the green investment cost to power three $\left(\text{i.e., } \frac{1}{2}\beta\theta^3\right)$. Adopting the similar method, the optimal green level, wholesale price, and retail price in the traditional reselling scheme, agency selling scheme and all the other shareholding strategies can be obtained. To save the space, only the observations and conclusions are provided in this part, while the optimal solutions and their monotonicity and comparisons are placed in Appendix E.

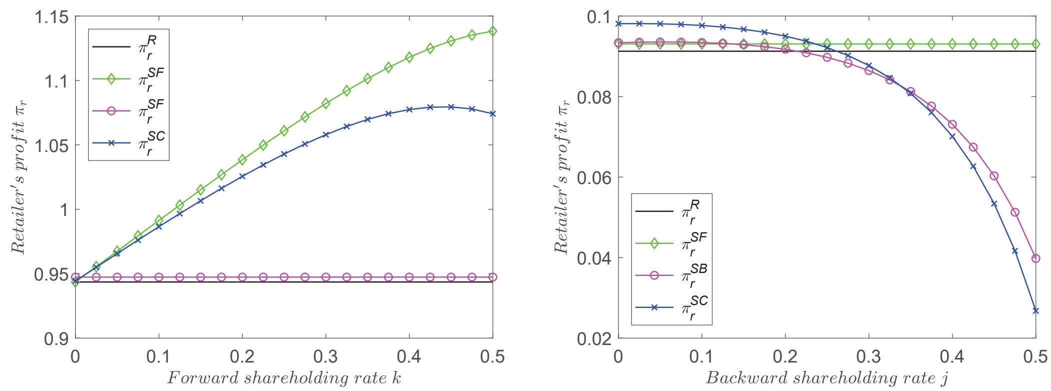
Thereby, for the shareholding strategy preferences of the participants, we first discuss the impacts of shareholding rates k and j on both player and the whole supply chain's profits and then investigate whether there exists Pareto-regions in this extended model. While the profit functions of the players are too complex and intractable, we propose a numerical example to display our results. Through the observations of Figure 7 and Figure 8, we find the shareholding rates' impacts on the participants' profits remain. On the one hand, a higher forward/backward shareholding rate always provides the supplier a higher profit. On the other hand, it shows that the retailer's profit first increase with shareholding rates and then decrease with them. These findings echo the results of the basic model and prove their robustness.



(a) Supplier's profits under different forward shareholding rates k

(b) Supplier's profits under different backward shareholding rates j

Figure 7: Supplier's profits under different forward and backward shareholding rates k and j



(a) Platform retailer's profits under different forward shareholding rates k

(b) Platform retailer's profits under different backward shareholding rates j

Figure 8: Platform retailer's profits under different forward and backward shareholding rates k and j

For the strategy preferences of the supplier and retailer, we observe that cross-shareholding strategy remains the supplier's optimal choice. Besides, the shareholding preference of the retailer is dependent on the shareholding structure as in the basic model. Specifically, when more than one shareholidng rate is sufficiently high, the retailer prefers forward shareholding. Similar to the results in the main body, when both shareholding rates are sufficiently low, the retailer will also choose cross-shareholding as its optimal strategy and the Pareto-region remains.

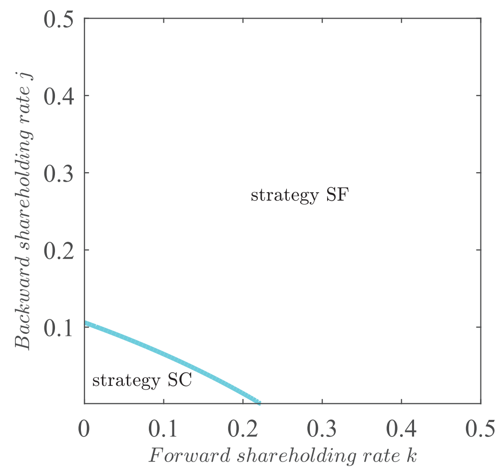


Figure 9: Shareholding strategy preferences of the platform retailer under different level of forward and backward shareholding rates k and j

8. Conclusions and Implications

8.1. Conclusions

In a platform supply chain that faces with green sensitive consumers, the upstream supplier will invest in the greenness of the products to improve the sales volume and through which increase its profits. However, the downstream platform retailer can gain more profits costless when the green level increases and can be regarded as a free-riding, which will conversely lead to the under-investment problem of the upstream. To address this issue, a novel coordination scheme—shareholding is proposed in this work. Forward and backward shareholding rates as well as three shareholding strategies including forward, backward and cross-shareholding strategy have been exploited. Under this approach, the supplier and the platform retailer can redistribute the wholesale and retailing stages' profits depending on the shares held by them. In the further investigation of the shareholding, holding share effects and incentive effects are derived in the shareholdings. Particularly, in forward shareholding strategy, through holding partial shares of the platform retailer, the supplier can access the market more directly and acquire the retailing profit to some extend in the form of holding share effects. Meanwhile, the retailer may also benefit from the progress through incentive effects while the supplier has the incentives to increase the profit in retailing. Similarly, in backward shareholding strategy, partial shares of the supplier is allocated to the retailer. The supplier incurs a loss in its shareholding but may benefit from the retailer's operation decisions. Specifically, the platform retailer will consider the profits in wholesale progress while making decisions and thereby the incentive effects arise. At last, in cross-shareholding strategy, the characters of both single-way shareholdings exist. Shareholding rates' interaction also appears under this shareholding strategy. Using the classic reselling scheme as the background, this work has examined the optimal green level, wholesale price, retail price and profits of the players while using different shareholding strategies.

The contributions of this study are three-fold: Firstly, we analyze the impacts of shareholding rates on the optimal green level and wholesale price of the supplier, and the optimal retail price of the retailer. For the optimal green level, it shows that the optimal green level is increases in response to either of the (forward or backward) shareholding rate under all shareholding strategies. Revealing the positive effect of shareholding on the coordination of the green level. Moreover, when adopting this method, promoting the retailer to hold the supplier's shares will be more capable than promoting the supplier's holding shares. Meanwhile, in observing the optimal wholesale price and retail price, we note that both the prices are increases in the backward shareholding rate and decreases in the forward one in most occasions. However, these two prices may change in opposite directions when the forward shareholding exceeds a certain threshold in using the cross-shareholding strategy. That is, when the supplier holds a sufficiently high partial of the retailer's share, and meanwhile it's partial share is held by the retailer, the holding share effect becomes less attractive and the supplier inclines to raise its profit in the wholesale progress. However, to stabilize the demand, the retailer can't adjust the retail price in the same way. It also indicates the pricing decision of supplier is more flexible than the retailer and the supplier's further extraction of the platform retailer's margin.

Secondly, the shareholding strategy preferences of the supplier and the platform retailer have been investigated. On the view of the supplier, the holding share effects can exert strictly positive effect on the supplier's performance for a positive forward shareholding rate, which can cover the negative impacts from less favorable operations of the retailer. For a higher backward shareholding rate, although loss part of its own profit, the retailer's more favorable operations as incentive effects can bring benefits to the supplier. Meanwhile, as the leader of the chain, the supplier can increase its wholesale price to possess a higher marginal and maintain the profit gains from the remaining share. Consequently, the supplier's profit is always increasing in both the forward and backward shareholding rates, leading cross-shareholding strategy to be its optimal shareholding strategy under any circumstance. On the view of the platform retailer, although it may suffer from a loss of shares in using the forward shareholding strategy, the incentive effects of a lower wholesale price and higher green level allows it to gain more profits overall, leading the platform retailer's optimal strategy to be the forward shareholding in most cases. However, when the forward shareholding rate continues to grow, the wholesale price becomes unfavorable and the marginal of the retailer is shrinked, resulting the retailer's optimal shareholding strategy converts to backward shareholding if the backward shareholding rate is low. In this situation, the holding share effects dominate the negative incentive effects, leading to an improvement of the retailer's profit. Moreover, when both shareholding rates are high, the retailer will not accept any shareholding strategy and choose to be totally independent. Interestingly, the retailer's optimal shareholding strategy changes to cross-shareholding when both the shareholding rates are sufficiently low. Under this occasion, both players of the chain will optimally choose the same strategy and thereby Pareto region goes with it. This conclusion helps explain the phenomena that many firms enjoy a low-level of the cross-shareholding in the real financial market like the Japanese one. Furthermore, under the alternative scheme of agency selling, cross-shareholding strategy remains their mutual optimal strategy if the agency sharing ratio is sufficiently high. Therefore, when the agency selling is not available in the practice, the related companies can consider to adopt the cross-shareholding to their performances.

Thirdly, from the perspective of the entire supply chain, whilst the increase in the forward shareholding rate always raises the aggregate profits, a higher backward shareholding rate may cause damages when it exceeds a certain level. Thus, if the upstream shareholding rate is high and downstream rate is medium, cross-shareholding strategy achieves and the aggregate profits of both parties reach its highest level. Comparing the chain's preference to both players, it shows the chain's highest favorable shareholding rates are larger than the retailer's. Consequently, if the supplier can compensate the retailer's holding share or being held in specific range of the shareholding rates, Pareto improvement can be achieved. This conclusion echoes some shareholding subsidy policies or stock options for the retailers provided by the suppliers (e.g., Haier and Luzhou Laojiao platform retailers) in the practice. Comparing with the agency selling scheme, the whole supply chain will acquire more profits under cross-shareholdng strategy when the agency sharing ratio is sufficiently high. We also check the robustness of our insights through introducing a specific form of endogenous share price and extending the green investment cost function to a more complicated one.

8.2. Practical implications

Based on the aforementioned conclusions, the practical implications can be proposed as follows. First, from view of the supply chain members, a supplier as the leader in a chain can improve its performance through increasing both forward and backward shareholding rates. Therefore, a supplier can encourage the backward shareholding of its retailer through a holding share compensation, or purchase the shares of its retailer at a premium to promote the forward shareholding. On the contrary, a retailer should prevent the supplier from holding too much of its share, and avoid holding much of its supplier's share in the meantime. Therefore, a low level of cross-shareholding is more proper for both supply chain members.

Second, from the view of the whole chain, the upper limits of both shareholding rates can be improved through appropriate adjustment on the share transactions. Besides, increasing the forward shareholding rate is more effective than the backward one in improving the whole supply chain. At the same time, as a higher shareholding rate does not always lead to a better performance (i.e., the chain's profit is concave to the backward shareholding rate), a supply chain should avoid blindly pursuing interest uniformity of the supply chain members (i.e., pursuing high forward and backward shareholding rates).

Third, from the view of the regulators or governments, green level and retail price can be both adjusted through shareholding structure's encouragement and regulation. On the one hand, as higher shareholding rates always lead to a higher green level, the regulators can encourage the enterprises to adopt shareholding scheme to lower the pollution. Meanwhile, increasing the backward shareholding rate is more effective than the forward one. On the other hand, as increasing the forward (backward) shareholding rate can raise (reduce) the retail price, the governor can also adjust the prices of the terminal market through the control of shareholding.

8.3. *Future research*

Whilst promising, this study can be further developed. Above all, both the green level and retail price are decided by the supplier under agency selling in our model, however, the retail price can be achieved by negotiation of the two parties. Besides, only the green investment of the upstream is considered while the downstream firm's investment efforts are ignored. However, an extension to address the investment of both parties is clearly desirable. In addition, although green level is considered to improve sales volume in this work, it may also exert a positive effect to the environment. When considering the environmental impact, one can also study the shareholding's impacts on the social welfare.

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Appendix A. Abbreviations

$$k_{w2} = \frac{1}{2(j-1)(a^2-2b\beta)}\left((-2j^2+2)b\beta + a^2(2j-1) - \sqrt{4b^2(j^2-6j+1)(j-1)^2\beta^2 + a^4\,(2j-1)^2 + 2a^2\,(2j^2-5j+2)\,(j-1)b\beta}\right)$$

$$k_{w3} = \frac{(6j^2-8j+2)b\beta + (2j-1)a^2}{(j-1)((4j-2)b\beta + a^2)}$$

$$k_1 = \frac{a}{4b(-2\beta(j-1)^2b + a^2j)\beta}\left((1-2j)a^3 + 4b(j^2+2j-1)\beta a + (4\beta(1-j)b - a^2)\sqrt{-(-8b\beta j^2 + a^2(4j-1))}\right)$$

$$k_2 = \frac{a}{4b(-2\beta(j-1)^2b + a^2j)\beta}\left((-2j+1)a^3 + 4b(j^2+2j-1)\beta a - (4\beta(1-j)b - a^2)\sqrt{-(-8b\beta j^2 + a^2(4j-1))}\right)$$

$$k_3 = \frac{a}{8\beta^2b^2(j-1)}\left((4(2j-1))\beta ba + (4b\beta - a^2)\sqrt{((j-1)^2a^2 - 8b\beta j^2)} + (-j+1)a^3\right)$$

$$k_4 = \frac{a}{8\beta^2b^2(j-1)}\left((-j+1)a^3 + (8(j-1/2))\beta ba - (4b\beta - a^2)\sqrt{((j-1)^2a^2 - 8b\beta j^2)}\right)$$

$$k_5 = \frac{1}{(8j-4)b\beta}\left(((12j-4)b\beta - a^2j) + \sqrt{16b^2(j-1)^2\beta^2 - 8a^2b(j^2-4j+2)\beta + (j^2-8j+4)a^4}\right)$$

$$k_6 = \frac{a^2(4b(j-1)\beta + a^2)(3j-1)}{2(-2b(j-1)^2\beta + a^2j)\beta(j-1)b}$$

Appendix B. Sketch maps of the other shareholding strategies

B.1. Backward shareholding strategy

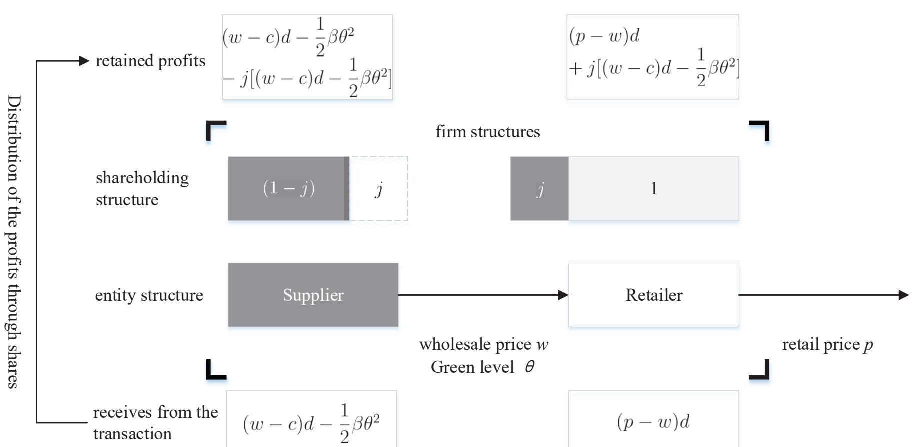

Figure B.10: Supply chain with backward shareholding

B.2. Cross-shareholding strategy

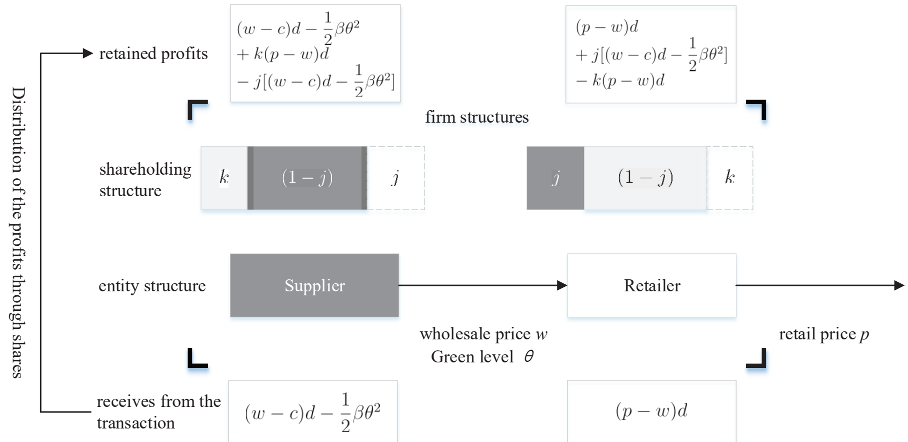

Figure B.11: Supply chain with cross-shareholding

Appendix C. Proofs of Propositions

Proof of Proposition 1

Before the discussion of the optimal solutions, we start with the Stackelberg game between both players. That is, as the follower of this game, we first derive the platform retailer's optimal retail price. While $\frac{\partial \pi_r^{R*}}{\partial p} = a\theta - bp + A - (p-w)b$ and $\frac{\partial^2 \pi_r^{R*}}{\partial p^2} = -2b < 0$, we get the optimal retail price of the retailer depending on the wholesale price and green level as $p^{R*} = \frac{a\theta + bw + A}{2b}$. Secondly, substituting this optimal retail price to the objective function of the Stackelberg leader (the supplier), the Hessian Matrix can be demonstrated as

$$\begin{Bmatrix} \frac{\partial^2 \pi_s^R}{\partial w^2} & \frac{\partial^2 \pi_s^R}{\partial w \partial \theta} \\ \frac{\partial^2 \pi_s^R}{\partial \theta \partial w} & \frac{\partial^2 \pi_s^R}{\partial \theta^2} \end{Bmatrix} = \begin{Bmatrix} -b & 1/2a \\ 1/2a & -\beta \end{Bmatrix},$$

which is absolutely negative definite. Adopting the simultaneous equations model (supplier's profit respective to the wholesale price and green level), we can obtain the optimal solutions of the supplier as follows.

$$\begin{cases} \theta^{R*} = \frac{a(A-bc)}{4b\beta - a^2}, \\ w^{R*} = \frac{-a^2c + 2b\beta c + 2A\beta}{4b\beta - a^2}. \end{cases}$$

Substituting the optimal decisions of the supplier to the retailer's, the retail price follows as

$$p^{R*} = \frac{(3A+bc)\beta - a^2c}{4b\beta - a^2}.$$

Thereby, the profit of each player can be obtained through the optimal solutions of the supplier and the retailer. Following the similar approach, we can obtain the optimal green level, wholesale

price, retail price and the players' performances under different shareholding strategies.
For the other strategies,

$$\begin{Bmatrix} \dfrac{\partial^2 \pi_s^{\mathrm{R}}}{\partial w^2} & \dfrac{\partial^2 \pi_s^{\mathrm{R}}}{\partial w \partial \theta} \\ \dfrac{\partial^2 \pi_s^{\mathrm{R}}}{\partial \theta \partial w} & \dfrac{\partial^2 \pi_s^{\mathrm{R}}}{\partial \theta^2} \end{Bmatrix} = \begin{Bmatrix} 1/2b(k-2) & 1/2a(1-k) \\ 1/2a(1-k) & 1(a^2k-2b\beta)/(2b) \end{Bmatrix}$$

We have $1(a^2k-2b\beta)/(2b)1/2b(k-2) - 1/2a(1-k)1/2a(1-k) = 1/2(2-k)b\beta - 1/4a^2 > 0$. Furthermore, we obsreve that the negative definite remains in all the shareholding strategies.
(i) Thereby, the green levels under various shareholding strategies' first partial derivatives with respective to the corresponding shareholding rates k and j can be obtained as

$$\frac{\partial \theta^{\mathrm{SF}*}}{\partial k} = \frac{2ab\beta(A-bc)}{\left(2b(2-k)\beta - a^2\right)^2}$$

and

$$\frac{\partial \theta^{\mathrm{SB}*}}{\partial j} = \frac{4ab\beta(A-bc)}{(4b(1-j)\beta - a^2)^2}.$$

Having the assumption of $(A-bc)>0$, it obtains $\frac{\partial \theta^{\mathrm{SF}*}}{\partial k} > 0$ and $\frac{\partial \theta^{\mathrm{SB}*}}{\partial j} > 0$.
In the cross-shareholding strategy, the optimal green level's first partial derivatives with respective to forward shareholding rate k and backward shareholding rate j are

$$\frac{\partial \theta^{\mathrm{SC}*}}{\partial k} = \frac{2ab\beta(A-bc)(1-j)}{\left(2\beta(k-2)(j-1)b - a^2\right)^2}$$

and

$$\frac{\partial \theta^{\mathrm{SC}*}}{\partial j} = \frac{2ab\beta(A-bc)(2-k)}{(2\beta(k-2)(j-1)b - a^2)^2},$$

respectively. Similarly, since $(A-bc)>0$, and $(1-j)>0$, $(2-k)>0$ for both $k<\frac{1}{2}$ and $j<\frac{1}{2}$, we can obtain that $\frac{\partial \theta^{\mathrm{SC}*}}{\partial k} > 0$ and $\frac{\partial \theta^{\mathrm{SC}*}}{\partial j} > 0$.
Meanwhile, to compare the impacts of the forward and backward shareholding rates on the green level, let $k=m=j$ and it follows by

$$\frac{\partial \theta^{\mathrm{SB}*}}{\partial m} - \frac{\partial \theta^{\mathrm{SF}*}}{\partial m} = \frac{\beta(m-1)^2(-2b(m-1)(m-3)\beta + a^2)(-bc+A)}{\left(m-\frac{1}{2}\right)\left(-2\beta(m-2)(m-1)b + a^2\right)^2}.$$

It can be observed that $2(m-1)(m-3) > \frac{1}{2}$, and combining the assumption $\frac{1}{2}b\beta > a^2$ in Section 3, we can get $\frac{\partial \theta^{\mathrm{SB}*}}{\partial m} > \frac{\partial \theta^{\mathrm{SF}*}}{\partial m}$. Similarly, we can obtain that $\frac{\partial \theta^{\mathrm{SC}*}}{\partial m} > \frac{\partial \theta^{\mathrm{SC}*}}{\partial m}$ through the above results.

(ii) We then compare the values of the optimal green levels under the benchmark and different

shareholding strategies. Firstly, the relationships between the optimal green level in each shareholding strategy and that in the benchmark are discussed.
Comparing the optimal green level in forward shareholding strategy shown in Section 4 with that in the reselling benchmark one in Section 3, we have

$$\theta^{\mathrm{SF}*} - \theta^{\mathrm{SR}*} = \frac{2ab\beta k(A-bc)}{(a^2-4b\beta)(2b(k-2)\beta+a^2)}.$$

Following the basic assumptions, it follows $(A-bc)$ is positive and $(a^2-4b\beta)$ is negative. Meanwhile, $2(2-k) > 3 > \frac{1}{2}$, then we can prove $\theta^{\mathrm{SF}*} > \theta^{\mathrm{R}*}$ is always valid.
Similarly, comparing the optimal green level in backward shareholding strategy with that in the reselling benchmark, it yields

$$\theta^{\mathrm{SB}*} - \theta^{\mathrm{R}*} = \frac{4ab\beta k(A-bc)}{(a^2-4b\beta)(4b(j-1)\beta+a^2)}.$$

It can be easily observed that $\theta^{\mathrm{SB}*} > \theta^{\mathrm{R}*}$. Similarly, we proved that $\theta^{\mathrm{SC}*} > \theta^{\mathrm{R}*}$.

Then we focus on the comparison of the optimal green levels under different shareholding cases. By comparing the optimal green levels in case SC and case SF, and then in case SC and case SB, we get

$$\theta^{\mathrm{SC}*} - \theta^{\mathrm{SF}*} = \frac{2ab\beta j(A-bc)(2-k)}{(2\beta(2-k)(1-j)b-a^2)(2b\beta(2-k)-a^2)}$$

and

$$\theta^{\mathrm{SC}*} - \theta^{\mathrm{SB}*} = \frac{2ab\beta k(A-bc)(1-j)}{(2\beta(2-k)(1-j)b-a^2)(2b\beta(1-j)-a^2)},$$

respectively, which are obviously positive. That is, $\theta^{\mathrm{SC}*} > \theta^{\mathrm{SF}*}$ and $\theta^{\mathrm{SC}*} > \theta^{\mathrm{SB}*}$.
At last, for the two single-way shareholding strategies, it follows

$$\theta^{\mathrm{SF}*} - \theta^{\mathrm{SB}*} = \frac{2b\beta(A-bc)(k-2j)a}{(4\beta(1-j)b-a^2)(2b(2-k)\beta-a^2)}.$$

While $A-bc>0$, and $4\beta(1-j)b-a^2<0$ and $2b(2-k)\beta-a^2<0$, the relationship between these two levels is dependent on $(k-2j)$. Thereby, if $(k-2j)>0$, $\theta^{\mathrm{SF}*} > \theta^{\mathrm{SB}*}$, otherwise $\theta^{\mathrm{SF}*} < \theta^{\mathrm{SB}*}$.

Proof of Proposition 2

(i) The impacts of shareholding rates on the optimal wholesale prices under different shareholding strategies are analyzed in this part. Firstly, the first partial derivative of $w^{\mathrm{SF}*}$ with respective to the forward shareholding rate k can be obtained as

$$\frac{\partial w^{\mathrm{SF}*}}{\partial k} = \frac{2\beta(A-bc)(a^2-2b\beta)}{(2b(k-2)\beta+a^2)^2}.$$

Notice that $(A-bc)>0$ and $(a^2-2b\beta)<0$, we have $\frac{\partial w^{\mathrm{SF}*}}{\partial k} < 0$.
Similarly, we get the first partial derivative of $w^{\mathrm{SB}*}$ with respective to the shareholding rate j

as

$$\frac{\partial w^{\mathrm{SB}*}}{\partial j} = \frac{8b\beta^2(A-bc)}{(4b(j-2)\beta+a^2)^2},$$

which satisfies $\frac{\partial w^{\mathrm{SB}*}}{\partial j} > 0$.
For the wholesale price under the cross-shareholding strategy, we can get

$$\frac{\partial w^{\mathrm{SC}*}}{\partial j} = \frac{2\beta(k-1)^2(A-bc)(2\beta(j-1)^2(2-k)b-a^2k)}{(-2\beta(k-2)(j-1)b+a^2)^2(j+k-1)^2} \tag{C.1}$$

and

$$\frac{\partial w^{\mathrm{SC}*}}{\partial k} = \frac{2\beta(k-1)(j-1)(A-bc)(\beta(j-1)((k-3)j-k+1)b-a^2(2j+k-1))}{(-2\beta(k-2)(j-1)b+a^2)^2(j+k-1)^2}. \tag{C.2}$$

For Eq. (C.1), $(1-k)$, $(1-j)$ and $(A-bc)$ are all positive. Besides, $\frac{2(j-1)^2(2-k)}{k} > \frac{3}{2} > \frac{1}{2}$, $(2\beta(j-1)^2(2-k)b-a^2k) > 0$. Then $\frac{\partial w^{\mathrm{SC}*}}{\partial j} > 0$ is always true.
For Eq. (C.2), we focus on

$$2\beta(k-1)(j-1)(A-bc)(\beta(j-1)((k-3)j-k+1)b-a^2(2j+k-1)), \tag{C.3}$$

which is increasing in k. Letting (C.3) $=0$, it yields if $k > \frac{(-6j^2+8j-2)b\beta-a^2(2j-1)}{-2\beta(j-1)^2b+a^2}$, $\frac{\partial w^{\mathrm{SC}*}}{\partial k} > 0$, otherwise $\frac{\partial w^{\mathrm{SC}*}}{\partial k} < 0$.

(ii) Comparing the wholesale prices under different shareholding strategies with that in the reselling benchmark, we can obtain that

$$w^{\mathrm{R}*} - w^{\mathrm{SF}*} = \frac{2\beta k(A-bc)(2b\beta-a^2)}{(a^2-4\beta(1-j)b)(a^2-4b\beta)},$$

$$w^{\mathrm{R}*} - w^{\mathrm{SB}*} = -\frac{8b\beta^2 j(A-bc)}{(a^2-4\beta b(1-j))(a^2-4b\beta)}$$

and

$$w^{\mathrm{R}*} - w^{\mathrm{SC}*} = \frac{2\beta(A-bc)((2(j-1))(-k^2+(j+1)k-2j)\beta b+ka^2(k(j-1)-2j+1))}{(a^2-4b\beta)(a^2-2b\beta(k-2)(j-1))(k+j-1)}, \tag{C.4}$$

respectively. Similar to the proofs of Proposition 1, it follows $2b\beta-a^2>0$ and $a^2-4\beta b(1-j)<0$ and thereby $w^{\mathrm{R}*} > w^{\mathrm{SF}*}$ and $w^{\mathrm{R}*} < w^{\mathrm{SB}*}$.
While since $(A-bc>0)$ and the denominator is positive as the assumption $a^2 < \frac{1}{2}b\beta$, the sign of Eq. (C.4) is dependent on

$$\eta_1 = -(2(j-1))(-k^2+(j+1)k-2j)\beta b+ka^2(k(j-1)-2j+1). \tag{C.5}$$

To discuss the whether η_1 is positive or negative, we first get its second derivative with respective to the shareholding rate k as $\frac{\partial^2 \eta_1}{\partial k^2} = 2(j-1)(a^2+2b\beta) < 0$. Then by solving $\eta_1 = 0$, we can derive

$$k = k_{w2(w2')} = \frac{1}{2(j-1)(a^2+2b\beta)}\Big((2j^2-2)b\beta + (2j-1)a^2 \pm \sqrt{4b^2(j^2-6j+1)(j-1)^2\beta^2 - (8(j^2-(5/2)j+1/2))a^2b(j-1)\beta + 4a^4(j-1/2)^2}\Big).$$

While the larger root is omitted as it is strictly larger than $\frac{1}{2}$. Thus, we get only one threshold k_{w2}. Then the comparison result between $w^{\mathrm{R}*}$ and $w^{\mathrm{SC}*}$ is obtained. If $k < k_{w2}$, $w^{\mathrm{R}*} < w^{\mathrm{SC}*}$, otherwise $w^{\mathrm{R}*} > w^{\mathrm{SC}*}$.

By comparing the wholesale price in the forward shareholding strategy with that the cross one, we can easily get

$$w^{\mathrm{SF}*} - w^{\mathrm{SC}*} = \frac{(2(-2\beta(k-2)(j-1)b + ka^2))(k-1)j(-bc+A)\beta}{(-2\beta(k-2)(j-1)b+a^2)(j+k-1)(2b(k-2)\beta+a^2)},$$

and prove that $w^{\mathrm{SF}*} < w^{\mathrm{SC}*}$.

Similarly, in backward shareholding strategy and cross-shareholding strategy, it has

$$w^{\mathrm{SB}*} - w^{\mathrm{SC}*} = \frac{k(-bc+A)\beta((4(j-1))((2k-3)j-k+1)\beta b + a^2((k-2)j-k+1))}{(4\beta(j-1)b+a^2)(-2\beta(k-2)(j-1)b+a^2)(j+k-1)}.$$

To discuss both wholesale prices' relationship, we focus on

$$\eta_2 = -(4(j-1))((k-3/2)j - (1/2)k + 1/2)\beta b - a^2((k-2)j-k+1).$$

Then we can get η_2's first partial derivative with respective to forward shareholding rate k is $\frac{\partial \eta_2}{\partial k} = (1-j)(4j-2)b\beta + a^2$. Solving $\eta_2 = 0$ yields

$$k = k_{w3} = \frac{(6j^2-8j+2)b\beta + a^2(2j-1)}{(j-1)((4j-2)b\beta+a^2)}.$$

Following the assumptions in Section 3, if $j > \frac{2b\beta - a^2}{4b\beta}$, $\frac{\partial \eta_2}{\partial k} > 0$, i.e., η_2 is increasing in k. If $j < \frac{2b\beta - a^2}{4b\beta}$, $\frac{\partial \eta_2}{\partial k} < 0$. However, $k_{w3} > \frac{1}{2}$ under this setting. Thereby, we have if $j < \frac{2b\beta - a^2}{4b\beta}$, $w^{\mathrm{SB}*} > w^{\mathrm{SC}*}$. if $j > \frac{2b\beta - a^2}{4b\beta}$, when $k < k_{w3}$, $w^{\mathrm{SB}*} > w^{\mathrm{SC}*}$, otherwise $w^{\mathrm{SB}*} < w^{\mathrm{SC}*}$.

Proof of Proposition 3

(i) The proof of Proposition 3 (i) is obviously and is quite similar to that of Proposition 1 (i) and Proposition 2 (i), we omit it.

(ii) Then the relationships between the optimal retail prices are discussed in this per proposition.

Firstly, comparing the retail prices under different shareholding strategies with that in the reselling benchmark yields

$$p^{\mathrm{R}*} - p^{\mathrm{SF}*} = \frac{2\beta k(b\beta - a^2)(-bc + A)}{(2b(k-2)\beta + a^2)(a^2 - 4b\beta)},$$

$$p^{\mathrm{R}*} - p^{\mathrm{SB}*} = -\frac{3a^2\beta j(-bc + A)}{(4\beta(j-1)b + a^2)(a^2 - 4b\beta)}$$

and

$$p^{\mathrm{R}*} - p^{\mathrm{SC}*} = \frac{2\beta(-bc + A)(-k\beta(j-1)b + a^2(k(j-1) - (3/2)j))}{(-2\beta(k-2)(j-1)b + a^2)(a^2 - 4b\beta)}, \tag{C.6}$$

respectively. We can easily conclude that $p^{\mathrm{R}*} > p^{\mathrm{SF}*}$ and $p^{\mathrm{R}*} < p^{\mathrm{SB}*}$. Meanwhile, since the sign of Eq. (C.6) is the same as that of

$$\eta_3 = -k\beta(j-1)b + a^2\left(k(j-1) - \frac{3}{2}j\right).$$

Therefore, if $\eta_3 > 0$, that is, $k > \dfrac{3a^2 j}{2(a^2 j - b\beta j - a^2 + b\beta)}$, $p^{\mathrm{R}*} > p^{\mathrm{SC}*}$, otherwise $p^{\mathrm{R}*} < p^{\mathrm{SC}*}$.
Then we compare the retail prices in different shareholding strategies and get

$$p^{\mathrm{SF}*} - p^{\mathrm{SB}*} = -\frac{3\beta(-bc + A)(-(2/3)k\beta(j-1)b + a^2(j - (2/3)k))}{(4\beta(j-1)b + a^2)(2b(k-2)\beta + a^2)}, \tag{C.7}$$

$$p^{\mathrm{SF}*} - p^{\mathrm{SC}*} = \frac{a^2\beta j(2k-3)(-bc + A)}{(-2\beta(k-2)(j-1)b + a^2)(2b(k-2)\beta + a^2)} \tag{C.8}$$

and

$$p^{\mathrm{SB}*} - p^{\mathrm{SC}*} = \frac{2\beta(-bc + A)(j-1)(\beta(j-1)b + a^2)k}{(4\beta(j-1)b + a^2)(-2\beta(k-2)(j-1)b + a^2)}. \tag{C.9}$$

For the sign of Eq. (C.7), we should discuss

$$\eta_4 = -(-\frac{2}{3}k\beta(j-1)b + a^2(j - \frac{2}{3}k)).$$

Since η_4's first derivative with respective to the forward shareholding rate k is $\dfrac{\partial \eta_4}{\partial k} = 2\beta(j - 1)b + 2a^2 < 0$, solving $\eta_4 = 0$ yields $k = \dfrac{3a^2 j}{2\beta(j-1)b + 2a^2} < 0$. Therefore, $p^{\mathrm{SF}*} - p^{\mathrm{SB}*} < 0$ is always valid.
For Eq. (C.8) and Eq. (C.9), it's obviously that both $p^{\mathrm{SF}*} < p^{\mathrm{SC}*}$ and $p^{\mathrm{SB}*} > p^{\mathrm{SC}*}$ always hold.

Proof of Proposition 4

(i) To prove Proposition 4 (i), it's easy to obtain that the first partial derivatives with respective to shareholding rates k and j of the supplier's profit are

$$\frac{\partial \pi_s^{\mathrm{SF}*}}{\partial k} = \frac{4b\beta^2(-bc + A)^2}{(4b(k-2)\beta + 2a^2)^2}$$

and

$$\frac{\partial \pi_s^{\mathrm{SB}*}}{\partial j} = \frac{(-bc+A)^2\beta a^2}{2(4\beta(j-1)b+a^2)^2}.$$

In the cross-shareholding strategy, we find

$$\frac{\partial \pi_s^{\mathrm{SC}*}}{\partial k} = \frac{(-bc+A)^2(j-1)^2\beta^2 b}{(-2\beta(k-2)(j-1)b+a^2)^2}$$

and

$$\frac{\partial \pi_s^{\mathrm{SC}*}}{\partial j} = \frac{(-bc+A)^2\beta a^2}{2(-2\beta(k-2)(j-1)b+a^2)^2}.$$

We can easily prove that the above equations are all positive, that is, the supplier's profit is positive correlated to shareholding rates k and j under all shareholding strategies.

(ii) In addition, for the platform retailer, we first discuss its profits in single-way shareholding strategies. Notice their first partial derivatives with respective to shareholding rates k and j are

$$\frac{\partial \pi_r^{\mathrm{SF}*}}{\partial k} = -\frac{(-bc+A)^2 b\beta^2(-2b\beta k+a^2)}{(2b(k-2)\beta+a^2)^3} \tag{C.10}$$

and

$$\frac{\partial \pi_r^{\mathrm{SB}*}}{\partial j} = -\frac{(-bc+A)^2\beta a^2(-8b\beta j+a^2)}{2(4\beta(j-1)b+a^2)^3}, \tag{C.11}$$

For Eq. (C.10), when $-2b\beta k+a^2>0$ (i.e., $k<\frac{a^2}{2b\beta}$), $\frac{\partial \pi_r^{\mathrm{SF}*}}{\partial k}>0$, otherwise $\frac{\partial \pi_r^{\mathrm{SF}*}}{\partial k}<0$.
For Eq. (C.11), when $-8b\beta j+a^2>0$ (i.e., $j<\frac{a^2}{8b\beta}$), $\frac{\partial \pi_r^{\mathrm{SB}*}}{\partial j}>0$, otherwise $\frac{\partial \pi_r^{\mathrm{SB}*}}{\partial j}<0$.
In the cross-shareholding strategy, we can get

$$\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial k} = -\frac{\beta^2(-bc+A)^2(2k\beta(j-1)^2 b+a^2(3j-1))(j-1)b}{(-2\beta(k-2)(j-1)b+a^2)^3} \tag{C.12}$$

and

$$\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial j} = -\frac{\beta(-bc+A)^2(2((3k-4)j-k)b\beta+a^2)a^2}{2(-2\beta(k-2)(j-1)b+a^2)^3}. \tag{C.13}$$

For Eq. (C.12), since the sign of $\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial k}$ is dependent on $\eta_5=-(2k\beta(j-1)^2b+a^2(3j-1))$ and η_5's first partial derivative respective to forward shareholding rate k is $\frac{\partial \eta_5}{\partial k}=-2\beta(1-j)^2b<0$. Therefore then we get if $k<\frac{a^2(1-3j)}{2\beta(j-1)^2 b}$, $\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial k}>0$, otherwise $\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial k}<0$.
For Eq. (C.13), similarly, its sign is the same as $\eta_6=2((3k-4)j-k)b\beta+a^2$, while it can be obtained that $\frac{\partial \eta_6}{\partial j}<0$. Thereby, if $j<\frac{2b\beta k-a^2}{2(3k-4)b\beta}$, $\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial j}>0$, otherwise $\frac{\partial \pi_r^{\mathrm{SC}*}}{\partial j}<0$.

Proof of Proposition 5

Firstly, we compare the profits of the supplier under different shareholding strategies with that in the reselling benchmark and get

$$\pi_s^{\mathrm{SF}*} - \pi_s^{\mathrm{R}*} = \frac{\beta^2(-bc+A)^2 bk}{(2b(k-2)\beta + a^2)(a^2 - 4b\beta)},$$

$$\pi_s^{\mathrm{SB}*} - \pi_s^{\mathrm{R}*} = \frac{\beta(-bc+A)^2 a^2 j}{2(4\beta(j-1)b + a^2)(a^2 - 4b\beta)}$$

and

$$\pi_s^{\mathrm{SC}*} - \pi_s^{\mathrm{R}*} = \frac{(-bc+A)^2(-2k(j-1)b\beta + a^2 j)\beta}{2(-2\beta(k-2)(j-1)b + a^2)(a^2 - 4b\beta)},$$

respectively. We can easily find that the above three equations are all positive and $\pi_s^{\mathrm{n}*} > \pi_s^{\mathrm{R}*}$ $(n \in \mathrm{SF},\ \mathrm{SB},\ \mathrm{SC})$ holds.

Then we compare the profit in cross-shareholding case with those of two single-way shareholdings and get

$$\pi_s^{\mathrm{SC}*} - \pi_s^{\mathrm{SF}*} = \frac{(-bc+A)^2 \beta a^2 j}{2(-2\beta(k-2)(j-1)b + a^2)(2b(k-2)\beta + a^2)}$$

and

$$\pi_s^{\mathrm{SC}*} - \pi_s^{\mathrm{SB}*} = \frac{(-bc+A)^2 (j-1)^2 \beta^2 bk}{(-2\beta(k-2)(j-1)b + a^2)(4\beta(j-1)b + a^2)}.$$

similarly, we can observe that $\pi_s^{\mathrm{SC}*} > \pi_s^{\mathrm{SF}*}$ and $\pi_s^{\mathrm{SC}*} > \pi_s^{\mathrm{SB}*}$.

At last, we compare the profits of supplier under two single-way shareholding strategies and get

$$\pi_s^{\mathrm{SB}*} - \pi_s^{\mathrm{SF}*} = \frac{(-bc+A)^2 \beta(2bk(j-1)\beta + a^2 j)}{2(2b(k-2)\beta + a^2)(4\beta(j-1)b + a^2)}. \tag{C.14}$$

We can easily prove that the sign of Eq. (C.14) is same to $(2bk(j-1)\beta + a^2 j)$. While its first partial derivative with respective to the forward shareholding rate k is $2b\beta(j-1) < 0$, we can prove that if $k < \dfrac{a^2 j}{2b\beta(1-j)}$, $\pi_s^{\mathrm{SB}*} > \pi_s^{\mathrm{SF}*}$, otherwise $\pi_s^{\mathrm{SB}*} < \pi_s^{\mathrm{SF}*}$.

Based on the description above, the supplier always inclines to the cross-shareholding among the three shareholding strategies.

Proof of Proposition 6

Firstly, we compare the profit of the platform retailer in the shareholding strategies with that in the reselling benchmark, it yields

$$\pi_r^{\mathrm{SF}*} - \pi_r^{\mathrm{R}*} = -\frac{b\beta^2(-bc+A)^2 k(4b^2\beta^2 k + a^4 - 4a^2 b\beta)}{(a^2 - 4b\beta)^2(2b(k-2)\beta + a^2)^2}, \tag{C.15}$$

$$\pi_r^{\mathrm{SB}*} - \pi_r^{\mathrm{R}*} = -\frac{\beta(16b^2\beta^2 j - 2a^2 b(j+2)\beta + a^4)(-bc+A)^2 j a^2}{2(a^2 - 4b\beta)^2(4\beta(j-1)b + a^2)^2} \tag{C.16}$$

and

$$\pi_r^{\mathrm{SC}*} - \pi_r^{\mathrm{R}*} = -\frac{1}{2(-2\beta(k-2)(j-1)b+a^2)^2(a^2-4b\beta)^2}\Big((8b^3k^2(j-1)^2\beta^3 - (16((k-1)j^2- (3/2)kj+(1/2)k))b^2a^2\beta^2 + 2b((k-1)j^2+(-2k-2)j+k)a^4\beta + a^6j)(-bc+A)^2\beta\Big). \tag{C.17}$$

For Eq. (C.15), it obviously that if $k < \dfrac{4a^2b\beta - a^4}{4b^2\beta^2}$, $\pi_r^{\mathrm{SF}*} > \pi_r^{\mathrm{R}*}$, otherwise $\pi_r^{\mathrm{SF}*} < \pi_r^{\mathrm{R}*}$. The same procedure may be easily adapted for Eq. (C.16) to obtain that if $j < \dfrac{a^4 - 4a^2b\beta}{2a^2b\beta - 16b^2\beta^2}$, $\pi_r^{\mathrm{SB}*} > \pi_r^{\mathrm{R}*}$, otherwise $\pi_r^{\mathrm{SB}*} < \pi_r^{\mathrm{R}*}$. At last, to discuss the sign of Eq. (C.17), we focus on

$$\eta_7 = -\Big(8b^3k^2(j-1)^2\beta^3 - (16((k-1)j^2-(3/2)kj+(1/2)k))b^2a^2\beta^2 + 2b((k-1)j^2+(-2k-2)j+k)a^4\beta + a^6j\Big).$$

While $\dfrac{\partial^2\eta_7}{\partial k^2} = -16b^3(j-1)^2\beta^3 < 0$ and solving $\eta_7 = 0$ yields

$$k_3 = \frac{a((-j+1)a^3 + (8(j-1/2))\beta ba + \sqrt{((j-1)^2a^2 - 8b\beta j^2)(a^2-4b\beta)^2})}{8b^2\beta^2(j-1)}$$

and

$$k_4 = \frac{a((-j+1)a^3 + (8(j-1/2))\beta ba - \sqrt{((j-1)^2a^2 - 8b\beta j^2)(a^2-4b\beta)^2})}{8b^2\beta^2(j-1)}.$$

We then discover that both k_3 and k_4 may exist in the interval of $\left(0,\ \dfrac{1}{2}\right)$. Therefore, if $k \in (k_3,\ k_4)$, $\pi_r^{\mathrm{SC}*} > \pi_r^{\mathrm{R}*}$, otherwise $\pi_r^{\mathrm{SC}*} < \pi_r^{\mathrm{R}*}$.

Secondly, comparing the the profits under the two single-way shareholding strategies yields

$$\pi_r^{\mathrm{SB}*} - \pi_r^{\mathrm{SF}*} = -\frac{1}{2(4\beta(j-1)b+a^2)^2(2b(k-2)\beta+a^2)^2}\Big((-bc+A)^2(-8b^3k^2(j-1)^2\beta^3 - (8((k-2)j^2+(-(1/2)k^2+2k)j-k))b^2a^2\beta^2 - 2b(j^2+(-2k+2)j+k)a^4\beta + a^6j)\beta\Big). \tag{C.18}$$

Herein, to analyze the sign of Eq. (C.18), we focus on

$$\eta_8 = -\Big(-8b^3k^2(j-1)^2\beta^3 - 8b^2a^2((k-2)j^2+(-(1/2)k^2+2k)j-k)\beta^2 - (2(j^2+(-2k+2)j+k))ba^4\beta + a^6j\Big).$$

While $\dfrac{\partial^2\eta_8}{\partial k^2} = -8b^2(-2\beta(j-1)^2b + a^2j)\beta^2 > 0$, and solving $\eta_8 = 0$ it follows

$$k_1 = \frac{a}{4b(-2\beta(j-1)^2b+a^2j)\beta}\Big((-2j+1)a^3 + 4b(j^2+2j-1)\beta a + \sqrt{-4(4\beta(j-1)b+a^2)^2(-2b\beta j^2+a^2(j-1/4))}\Big)$$

and

$$k_2 = \frac{a}{4b(-2\beta(j-1)^2 b + a^2 j)\beta}\Big((-2j+1)a^3 + 4b(j^2+2j-1)\beta a - \sqrt{-4(4\beta(j-1)b+a^2)^2(-2b\beta j^2 + a^2(j-1/4))}\Big).$$

Similarly, both k_1 and k_2 lie in $\left(0,\ \frac{1}{2}\right)$. Then if $k_1 < k < k_2$, $\pi_r^{\mathrm{SB}*} < \pi_r^{\mathrm{SF}*}$, otherwise $\pi_r^{\mathrm{SB}*} > \pi_r^{\mathrm{SF}*}$.

Ultimately, comparing the profit in cross-shareholding strategy with those in the two single-way shareholdings, we can obtain

$$\pi_r^{\mathrm{SC}*} - \pi_r^{\mathrm{SF}*} = -\frac{(-bc+A)^2 j\beta a^2}{2(-2\beta(k-2)(j-1)b+a^2)^2(2b(k-2)\beta+a^2)^2}\Big((8(k-2))b^2((k-1)j - (1/2)k)\beta^2 + 2b(-2+(k-1)j)a^2\beta + a^4\Big) \tag{C.19}$$

and

$$\pi_r^{\mathrm{SC}*} - \pi_r^{\mathrm{SB}*} = \frac{3(1-j)(-bc+A)^2 kb\beta^2}{(-2\beta(k-2)(j-1)b+a^2)^2(4\beta(j-1)b+a^2)^2}\Big((4/3)k\beta^2(j-1)^3b^2 + a^4(j-1/3) - (2/3(j-1))((k-6)j+2)\beta a^2 b\Big), \tag{C.20}$$

respectively.
To simplify the analysis of Eq. (C.19) and Eq. (C.20), we focus on

$$\eta_9 = -((8(k-2))b^2((k-1)j-(1/2)k)\beta^2 + 2b(-2+(k-1)j)a^2\beta + a^4)$$

and

$$\eta_{10} = (4/3)k\beta^2(j-1)^3b^2 + a^4(j-1/3) - (2/3(j-1))((k-6)j+2)\beta a^2 b.$$

While $\frac{\partial^2\eta_9}{\partial k^2} = (-16j+8)b^2\beta^2 > 0$, by $\eta_9 = 0$, we can get

$$k_5 = \frac{\sqrt{16b^2(j-1)^2\beta^2 - 8a^2b(j^2-4j+2)\beta + (j^2-8j+4)a^4} + (12j-4)b\beta - ja^2}{(8j-4)b\beta}$$

and

$$k_5' = \frac{-\sqrt{16b^2(j-1)^2\beta^2 - 8a^2b(j^2-4j+2)\beta + (j^2-8j+4)a^4} + (12j-4)b\beta - ja^2}{(8j-4)b\beta}.$$

As further observe that $k_5' > \frac{1}{2}$ and is invalid in the proposed model, k_5' is omitted in the discussion. Therefore, if $k > k_5$, $\pi_r^{\mathrm{SC}*} < \pi_r^{\mathrm{SF}*}$, otherwise $\pi_r^{\mathrm{SC}*} > \pi_r^{\mathrm{SF}*}$.
Notice that

$$\frac{\partial\eta_{10}}{\partial k} = \left(\frac{2}{3}(-2b(j-1)^2\beta + ja^2)\right) b(1-j)\beta < 0.$$

By solving $\eta_{10} = 0$, we can derive

$$k = k_6 = \frac{((4b(j-1)\beta + a^2)a^2)(3j-1)}{2(-2b(j-1)^2\beta + ja^2)b(j-1)\beta}.$$

Thus, if $k < k_6$, $\pi_r^{\mathrm{SC}*} > \pi_r^{\mathrm{SB}*}$, otherwise $\pi_r^{\mathrm{SC}*} < \pi_r^{\mathrm{SB}*}$.

Proof of Corollary 1

By substituting for $\frac{a^2}{b\beta}$ with t, the thresholds in Propositions 6 can be displayed as follows.

$$k_3 = \frac{(-t^2+8t)j - (t-4)(-t+\sqrt{t((j-1)^2t-8j^2)})}{8j-8},$$

$$k_4 = \frac{(-t^2+8t)j + (t-4)(t+\sqrt{t((j-1)^2t-8j^2)})}{8j-8},$$

$$k_5 = \frac{\sqrt{(t-4)^2j^2 - 8(t-2)^2j + 4(t-2)^2} - 4 + (-t+12)j}{8j-4},$$

$$k_6 = \frac{t(4j-4+t)(1-3j)}{2(2j^2-jt-4j+2)(j-1)}.$$

We then focus on the value of k_6 and observe that only when $j < \frac{1}{3}$ can lead a positive value of k_6. Then we discuss the relationship between k_6 and k_3 and find $k_6 - k_3 > 0$ is always valid when $t \in \left(0,\ \frac{1}{2}\right)$ and $j \in \left(0,\ \frac{1}{3}\right)$. At last, we compare k_5 and k_3. We find if $j < \frac{t(t-4)}{2t-16}$, $k_5 - k_3 > 0$. Combining the the above investigations, we further discover that $\frac{t(t-4)}{2t-16} < \frac{1}{3}$. On the whole, if $j < \frac{t(t-4)}{2t-16}$, there exists a nonempty set $K(j)$ that $k \in (k_3,\ \min\{k_4,\ k_5,\ k_6\})$ and in which the platform retailer will gain most profits in cross-shareholding.

Proof of Corollary 2

Based on the discussions in Propositions 5 and 6, both the players will gain more profits when k and j are in certain levels. In other words, Pareto area herein can be achieved.

Proof of Proposition 7

For the impacts of shareholding rates k and j on the performance of the whole supply chain, we can get

$$\frac{\partial \Pi^{\mathrm{SC}}}{\partial k} = -\frac{\beta(A-bc)^2(a^2+2b\beta(2k-3)(1-j)^2)}{2(a^2-2\beta b(2-k)(1-j))^2}$$

and

$$\frac{\partial \Pi^{\mathrm{SC}}}{\partial j} = -\frac{2a^2b\beta^2(A-bc)^2(2jk-3j-k+1)}{2(a^2-2\beta b(2-k)(1-j))^3}.$$

The former equation is definitely positive and the sign of the latter one is the same as that of

$(2jk-3j-k+1)$. Then if $j > \frac{k-1}{3-2k}$, $\frac{\partial \Pi^{\mathrm{SC}}}{\partial j} > 0$, otherwise $\frac{\partial \Pi^{\mathrm{SC}}}{\partial j} < 0$.

Proof of Corollary 3

Recall the platform retailer will benefit from the cross-shareholding if $k < \frac{8b\beta j - a^2}{(6j-2)b\beta}$, which is smaller than $\frac{1}{2}$. Therefore, the supplier can pay a positive price for the retailer's share and increase the forward shareholding rate to its upper limit.

Proof of Corollary 4

Recall the platform retailer will benefit from the cross-shareholding if $j < \frac{2b\beta k - a^2}{2(3k-4)b\beta}$, it's easy to prove that $\frac{k-1}{3-2k} > \frac{2b\beta k - a^2}{2(3k-4)b\beta}$ combining with the whole chain's preference to the shareholdings in Proposition 7. As a result, the supplier can compensate the backward shareholding of the retailer under this circumstance $\left(\text{i.e., } \frac{2b\beta k - a^2}{2(3k-4)b\beta} < j < \frac{k-1}{3-2k}\right)$ to incentivize a higher backward shareholding rate.

Proof of Proposition 8

Adopting the similar methods to the benchmark and shareholding models, the profits of the supplier and the retailer can be obtained. For the impacts of agency sharing ratio on the supplier's profit, it has $\frac{\partial \pi_s^{\mathrm{A}*}}{\partial x} = -\frac{b(Ax+bc-A)\beta(-c(-1+x)a^2+\beta(Ax-bc-A))}{((-1+x)a^2+2b\beta)^2(-1+x)^2}$. As the bottom of this function is a square, it only needs to discuss the numerator of this function. Analyzing this numerator, we observe that it's second derivative respective to the sharing ratio x is $-2A(A\beta - a^2c)b\beta < 0$. Solving $\frac{\partial \pi_s^{\mathrm{A}*}}{\partial x} = 0$, we get $x = \frac{A-bc}{A}, \frac{A\beta - a^2c + bc\beta}{A\beta - a^2c}$. While the second solution is larger than 1, we have $x = \frac{A-bc}{A}$. Therefore, if $0 < x < \frac{A-bc}{A}$, the supplier's profit is decrease to the sharing ratio, and when $\frac{A-bc}{A} < x < 1$, $\frac{\partial \pi_s^{\mathrm{A}*}}{\partial x} > 0$.
Then we similarly obtain $\frac{\partial \pi_r^{\mathrm{A}*}}{\partial x} = -\frac{1}{(-1+x)^3((-1+x)a^2+2b\beta)^3}\Big(b\beta(-c(-1+x)^2(Ax^2+2bcx+bc-A)a^4+(-1+x)\beta(A^2x^3+(2Abc-A^2)x^2+(-3b^2c^2-4Abc-A^2)x+(3bc+A)(-bc+A))a^2-2b\beta^2(A^2x^3-3A^2x^2+(b^2c^2+3A^2)x+b^2c^2-A^2))\Big)$. Following our assumptions in Section 3, we have $(-1+x) < 0$ and $(-1+x)a^2+2b\beta > 0$. Then the sign of $\frac{\partial \pi_r^{\mathrm{A}*}}{\partial x}$ is consistent with $\gamma_2 = -c(-1+x)^2(Ax^2+2bcx+bc-A)a^4+(-1+x)\beta(A^2x^3+(2Abc-A^2)x^2+(-3b^2c^2-4Abc-A^2)x+(3bc+A)(-bc+A))a^2-2b\beta^2(A^2x^3-3A^2x^2+(b^2c^2+3A^2)x+b^2c^2-A^2$. Firstly, we have $\frac{\partial \gamma_2^4}{\partial x^4} = -24Aa^4c + 24A^2a^2\beta = 24Aa^2(A\beta - a^2c) > 0$. Then $\frac{\partial \gamma_2^3}{\partial x^3}$ is increasing in (0, 1). Solve $\frac{\partial \gamma_2^3}{\partial x^3} = 0$, we get $x = \frac{(a^2\beta + b\beta^2)A^2 - a^2c(a^2+b\beta)A + c^2a^4b}{2Aa^2(-a^2c+A\beta)}$. Discussing the numerator and the denominator of this function, we find $(a^2\beta+b\beta^2)A^2-a^2c(a^2+b\beta)A+c^2a^4b-2Aa^2(-a^2c+A\beta) =$

$(-a^2\beta + b\beta^2)A^2 + a^2c(a^2 - b\beta)A + c^2a^4b = (b\beta - a^2)(A^2\beta - Aa^2c) + c^2a^4b > 0$. Therefore, we get $\frac{\partial\gamma_2^3}{\partial x^3} < 0$ in the interval of $(0,\ 1)$. Then it satisfies $\frac{\partial\gamma_2^2}{\partial x^2}$ decreasing in $(0,\ 1)$. Solve $\frac{\partial\gamma_2^2}{\partial x^2} = 0$, it follows $x_4 = \frac{1}{2Aa^2(-a^2c + A\beta)}\Big((bc^2 - Ac)a^4 + A\beta(-bc + A)a^2 + \beta^2A^2b \pm \sqrt{\beta^2(a^2 - b\beta)^2A^4 - 2a^2c\beta(a^2 - b\beta)^2A^3 + a^4c^2(a^4 - 2a^2b\beta + 5b^2\beta^2)A^2 - 4Aa^6b^2\beta c^3 + a^8b^2c^4}\Big)$, the larger root is dropped while $\frac{\partial\gamma_2^2}{\partial x^2}$ decreasing in its domain. We observe that $0 < ((bc^2 - Ac)a^4 + A\beta(-bc + A)a^2 + \beta^2A^2b)^2 - \Big(\beta^2(a^2 - b\beta)^2A^4 - 2a^2c\beta(a^2 - b\beta)^2A^3 + a^4c^2(a^4 - 2a^2b\beta + 5b^2\beta^2)A^2 - 4Aa^6b^2\beta c^3 + a^8b^2c^4\Big) < 1$. Then the first partial derivate of γ_2 respective to x is increasing in $(0,\ x_4)$ and decreasing in $(x_4,\ 1)$. Bringing $x = 0$, $x = 1$ and $x = x_4$ respectively into $\frac{\partial\gamma_2}{\partial x}$, it receives $\frac{\partial\gamma_2}{\partial x}$'s value at $(-2b^3c^2 - 6A^2b)\beta^2 + 2Aa^2(3bc + A)\beta - 2Aa^4c < 0$, $-\beta b^2(6a^2 + 2b\beta)c^2 < 0$ and $\frac{1}{A^2a^4(-a^2c + A\beta)^2}\Big((\beta^2(a^2 - b\beta)^2A^4 - 2a^2c\beta(a^2 - b\beta)^2A^3 + a^4c^2(a^4 - 2a^2b\beta + 5b^2\beta^2)A^2 - 4Aa^6b^2\beta c^3 + a^8b^2c^4)^(3/2) + \beta^3(a^2 - b\beta)^3A^6 - 3a^2c\beta^2(a^2 - b\beta)^3A^5 + 3c^2a^4(a^6 - 3a^4b\beta + 3a^2b^2\beta^2 - (11/3)b^3\beta^3)\beta A^4 - a^6c^3(a^6 - 3a^4b\beta + 3a^2b^2\beta^2 - 17b^3\beta^3)A^3 - 14A^2a^8b^3\beta^2c^4 + 6Aa^10b^3\beta c^5 - a^12b^3c^6\Big)$, respectively. Thereby, there exists at most two roots x_5 and x_6 to let $\frac{\partial\gamma_2}{\partial x} = 0$ in the interval of $(0,\ 1)$. Then we obtain that, $\frac{\partial\pi_r^{A*}}{\partial x}$ declines with x in the intervals of $(0,\ x_5)$ and $(x_6,\ 1)$ while inclines with x in the range of $(x_5,\ x_6)$. Bring $x = 0$ and $x = 1$ in γ_2, we get $\gamma_2 = (2b\beta - a^2)(-bc + A)((bc + A)\beta - a^2c) > 0$ and $\gamma_2 = -4b^3\beta^2c^2 < 0$, respectively. Besides, $\gamma_2 < 0$ if $x = x_6$. Then there exists an unique x_3 ($x_3 \in (x_5,\ x_6)$) let $\gamma_2 = 0$ in the range of $(0,\ 1)$. When $x \in (0,\ x_3)$, $\frac{\partial\pi_r^{A*}}{\partial x} > 0$, otherwise $\frac{\partial\pi_r^{A*}}{\partial x} < 0$. x_3 satisfies $RoofOf((-Aa^4c + A^2a^2\beta)\_Z^4 + (-2a^4bc^2 + 2Aa^4c + 2Aa^2b\beta c - 2A^2a^2\beta - 2A^2b\beta^2)\_Z^3 + (3a^4bc^2 - 3a^2b^2\beta c^2 - 6Aa^2b\beta c + 6A^2b\beta^2)\_Z^2 + (-2b^3\beta^2c^2 - 2Aa^4c + 6Aa^2b\beta c + 2A^2a^2\beta - 6A^2b\beta^2)\_Z - a^4bc^2 + 3a^2b^2\beta c^2 - 2b^3\beta^2c^2 + Aa^4c - 2Aa^2b\beta c - A^2a^2\beta + 2A^2b\beta^2)$ where Z is the placeholder.

Proof of Proposition 9

For the preferences of the supplier and the retailer to the agency selling and shareholding, we first discuss the supplier's profit. Solving $\pi_s^{SC*} - \pi_s^{A*} = 0$, we get $x_s = (\pm(((j - 1)(b^2c^2 - 4Ab(k - 3/2)c + A^2)\beta^2 + 2ca^2(b(k - 2)(j - 1)c + A)\beta - a^4c^2)(j - 1)b^2)^{1/2} + c\beta(j - 1)b^2 - (2(j - 1))(A(k - 3/2)\beta + (1/2)ca^2)b + Aa^2j)(-bc + A)/(a^2c^2(j - 1)b^2 - (2(j - 1))A(\beta(k - 2)A + ca^2)b + A^2a^2j)$, where the larger root can be omitted. Besides, it has $\frac{\partial(\pi_s^{SC*} - \pi_s^{A*})}{\partial x} = \frac{\beta(-c(-1 + x)a^2 + \beta(Ax - bc - A))b(Ax + bc - A)}{(-1 + x)^2((-1 + x)a^2 + 2b\beta)^2}$. Moreover, let $\eta_{11} = \beta(-c(-1 + x)a^2 + \beta(Ax - bc - A))b(Ax + bc - A)$, we have $\frac{\partial^2\eta_{11}}{\partial x^2} = 2A^2\beta - 2Aa^2c > 0$ and $\frac{\partial\eta_{11}}{\partial x}\Big|_{x=1} = -a^2bc^2 < 0$. Solve $\eta_{11} = 0$, we get $x = \frac{A - bc}{A}$. Thereby, if $x < x_s$, the supplier's profit under agency selling is higher than the cross-shareholding, otherwise the cross-shareholding is better. For the retailer's preference, we discuss the value between $\pi_r^{\mathrm{SC}*}$ and $\pi_r^{\mathrm{A}*}$. Equal to Proposition 8, $\pi_r^{\mathrm{SC}*} - \pi_r^{\mathrm{A}*}$ is decreasing in $(0,\ x_3)$ and increasing in $(x_3,\ 1)$. Solving $\pi_r^{\mathrm{SC}*} - \pi_r^{\mathrm{A}*} = 0$, we have x_{r1} and x_{r2} in the interval of $(0,\ x_3)$ and $(x_3,\ 1)$, respectively. In conclusion, when $x < x_{r1}$ or $x > x_{r2}$, cross-shareholding is more preferable to the retailer, otherwise agency selling is more profitable. Notice that x_{r1}, x_{r2} and x_c (in Proposition 10) are too complex to obtain analytic solutions, we only use the above characters to describe our results. Moreover, numerical example is presented to show the existence of our conclusion.

Proof of Corollary 5

Combining Propositions 8 and 9, we observe that when $x_s < x_{r_2} < 1$, both the players will choose cross-shareholding under the alternative scheme of agency selling.

Proof of Proposition 10

On the view of the whole supply chain, we first have $\frac{\partial \pi_c^{\mathrm{A}}}{\partial x} = \frac{2xb\beta}{((-1+x)a^2+2b\beta)^3(1-x)^3}$ $\Big(- 2b^3\beta^2c^2 - (Ax + (3/2)cb - A)c(-1+x)^2a^4 + \beta(-1+x)(A^2x^2 - 3b^2c^2 - 2A^2x + A^2)a^2\Big)$. Let $\eta_{12} = -(Ax + (3/2)cb - A)c(-1+x)^2a^4 + \beta(-1+x)(A^2x^2 - 3b^2c^2 - 2A^2x + A^2)a^2$, we have $\frac{\partial^3 \eta_{12}}{\partial x^3} = 6a^2A(-a^2c + A\beta) > 0$. Besides, $\left.\frac{\partial^2 \eta_{12}}{\partial x^2}\right._{x=1} < 0$. Then $\frac{\partial \eta_{12}}{\partial x}$ is decreasing in $(0,\ 1)$. While $\frac{\partial \eta_{12}}{\partial x} = 0$ when $x = \frac{A - bc}{A}$, η_{12} is increasing in $(0,\ \frac{A-bc}{A})$. Solve $\eta_{12} = 0$, we get $x_c = (1/2)(((bc^2 - 2Ac)a^3 + 2A^2a\beta)(c^2(a^8c^4 - 6Aa^6\beta c^3 + 14A^2a^4\beta^2c^2 - 16A^3a^2\beta^3c + 8A^4\beta^4 - 2A\sqrt{(-a^2c+2A\beta)^4}a^2\beta c + 2A^2\sqrt{(-a^2c+2A\beta)^4}\beta^2)a)^{1/3} + (a^6c^4 - 4Aa^4c^3\beta + 4A^2a^2\beta^2c^2 + (c^2(a^8c^4 - 6Aa^6\beta c^3 + 14A^2a^4\beta^2c^2 - 16A^3a^2\beta^3c + 8A^4\beta^4 - 2A\sqrt{(-a^2c+2A\beta)^4}a^2\beta c + 2A^2\sqrt{(-a^2c+2A\beta)^4}\beta^2)a)^{2/3})b)/((c^2(a^8c^4 - 6Aa^6\beta c^3 + 14A^2a^4\beta^2c^2 - 16A^3a^2\beta^3c + 8A^4\beta^4 + 2A^2\sqrt{(-a^2c+2A\beta)^4}\beta^2)a)^{1/3}A(-a^2c + A\beta)a) - 2A\sqrt{(-a^2c+2A\beta)^4}a^2\beta c$. When $x < x_c$, the chain's profit is decreasing to the agency sharing ratio, otherwise the profit will increase to it.

Therefore, we can easily obtain that $(\pi_c^{\mathrm{SC}*} - \pi_c^{\mathrm{A}*})$ is increasing to the agency sharing ratio. Meanwhile, solving $\pi_c^{\mathrm{SC}*} - \pi_c^{\mathrm{A}*} = 0$, we get x_c. Therefore, if $x < x_c$, agency selling performs better than cross-shareholding, otherwise cross-shareholding is better.

Proof of Proposition 11

Under considering of the share price, we then discuss the shareholding preferences of both players. For example, under the forward shareholding strategy, while $\pi_s^{\mathrm{SF}'} = (w-c)d - (1/2)\beta\theta^2 - kb\beta^2(-bc+A)^2/(a^2-4b\beta)^2 + k(p-w)d$ and $\pi_r^{\mathrm{SF}'} = (1-k)(p-w)d + kb\beta^2(-bc+A)^2/(a^2-4b\beta)^2$. The retailer's profit's second-order partial derivative respective to p is negative, and we get $p = (ak\theta + bkw + Ak - a\theta - bw - A)/(2bk - 2b)$ through solving $\frac{\partial \pi_r^{\mathrm{SF}'}}{\partial p} = 0$. Substituting this mediate function to the supplier's profit, we have $\pi_s^{\mathrm{SF}'} = (w-c)(a\theta - b(ak\theta + bkw + Ak - a\theta - bw - A)/(2bk - 2b) + A) - (1/2)\beta\theta^2 - kb\beta^2(-bc + A)^2/(a^2 - 4b\beta)^2 + k((ak\theta + bkw +$

$Ak - a\theta - bw - A)/(2bk - 2b) - w)(a\theta - b(ak\theta + bkw + Ak - a\theta - bw - A)/(2bk - 2b) + A)$. While its Hassian matrix is negative definite, the optimal solutions of the supplier are obtained as $\theta^{\mathrm{SF}'*} = -(-bc+A)a/(2b\beta k+a^2-4b\beta)$ and $w^{\mathrm{SF}'*} = (2A\beta k+a^2c-2b\beta c-2A\beta)/(2b\beta k+a^2-4b\beta)$. Similarly, we can obtain the profits of both players under backward shareholding and cross-shareholding. Based on the above outcomes, we propose the following discussions.

(i) Analyzing the impacts of forward shareholding rate on the players' profits, we draw the following conclusions. Under different level of forward shareholding rate, we have $\pi_s^{\mathrm{SF}'*} = -(1/2)(-bc+A)^2\beta(4\beta^2(k^2-2k+4)b^2+2a^2\beta(k-4)b+a^4)/((2\beta(k-2)b+a^2)(a^2-4b\beta)^2)$ and $\pi_r^{\mathrm{SF}'*} = (-bc+A)^2b(4\beta^2(k^3-4k^2+4)b^2+4a^2\beta(k^2-2)b+a^4)\beta^2/((2\beta(k-2)b+a^2)^2(a^2-4b\beta)^2)$. Discussing the monotonicity of both functions, we observe that $\dfrac{\partial \pi_s^{\mathrm{SF}'*}}{\partial k} = -(4(\beta(k-4)b+a^2))b^2k(-bc+A)^2\beta^3/((a^2-4b\beta)^2(2\beta(k-2)b+a^2)^2) > 0$ and $\dfrac{\partial \pi_r^{\mathrm{SF}'*}}{\partial k} = (8(\beta^2(k^3-6k^2+16k-8)b^2+(3/2)a^2(k^2-(16/3)k+8/3)\beta b+a^4(k-1/2)))b^2(-bc+A)^2\beta^3/((a^2-4b\beta)^2(2\beta(k-2)b+a^2)^3) > 0$ always valid in the domain of k. Then we can obtain that both players prefer a higher forward shareholding rate and will achieve the highest level of forward shareholding rate 1/2.

(ii) Regarding the backward shareholding rate, similarly, we get the profits of the supplier and the retailer as $\pi_s^{\mathrm{SB}'*} = -(1/2)((4j^2-4)\beta b+a^2)\beta(-bc+A)^2/((4\beta(j-1)b+a^2)(a^2-4b\beta))$ and $\pi_r^{\mathrm{SB}'*} = (5((8/5)(j-1)^2(j-1/2)\beta b+a^2(j^2-(4/5)j+1/5)))b\beta^2(-bc+A)^2/((a^2-4b\beta)(4\beta(j-1)b+a^2)^2)$, respectively. For the supplier, $\dfrac{\partial \pi_s^{\mathrm{SB}'*}}{\partial j} = -4b\beta^2(-bc+A)^2a^2/(4\beta(j-1)b+a^2)^3 > 0$. For the retailer, $\dfrac{\partial \pi_r^{\mathrm{SB}'*}}{\partial j} = 10(-bc+A)^2((16/5)\beta^2(j-1)^3b^2+(12/5)a^2(j^2-(8/3)j+1)\beta b+a^4(j-2/5))b\beta^2/((a^2-4b\beta)(4\beta(j-1)b+a^2)^3)$, which can be simplify to the problem of $\eta_{14} = (16/5)\beta^2(j-1)^3b^2+(12/5)a^2(j^2-(8/3)j+1)\beta b+a^4(j-2/5)$. Having $a^2 < \dfrac{1}{2}b\beta$, we can easily prove that $\eta_{14} < 0$. Combining with both monotonicity, it noticed that, although the supplier has strong incentive to increase the backward shareholding rate, the retailer always get a loss in this procedure, and thereby the backward shareholding rate is zero.

(iii) Under cross shareholding, it can be discovered that the observations in (i) and (ii) remain. That is, the forward shareholding rate reaches 1/2 and the backward shareholding rate remains zero.

Proof of Proposition 12

While only the price of supplier's share is adjusted and other properties in our model remain unchanged, we focus on the achievable of backward shareholding rate. For the supplier, it's profit converts to

$$\pi_s^{\mathrm{SB}'} = ((w-c)(a\theta - bp + A) - (1/2)\beta\theta^2)(1-j) - j\delta\beta(-bc+A)^2/(2a^2-8b\beta),$$

while the retailer's profit is replaced by

$$\pi_r^{\mathrm{SB}'} = ((w-c)(a\theta - bp + A) - (1/2)\beta\theta^2)(1-j) - j\delta\beta(-bc+A)^2/(2a^2-8b\beta).$$

Adopting the similar method, the optimal green level, wholesale price and retail price can be easily obtained and also the corresponding profits of both players.

(i) From the perspective of the supplier, while we have proved that the supplier prefers a higher backward shareholding under all strategies even without the share's payment from the retailer (i.e., $\delta = 0$), the existing conclusions remain unchanged.

(ii) From the perspective of the retailer, we have its profit's monotonicity to the backward shareholding rate j is $\frac{\partial \pi_r^{\mathrm{SB}'}}{\partial j} = (1/2)(64b^3\delta(j-1)^3 beta^3 + (48((j-1)^2\delta - (2/3)j))b^2a^2\beta^2 + (12((j-1)\delta + (2/3)j + 1/3))ba^4\beta + a^6(\delta-1))(-bc+A)^2\beta/((a^2-4b\beta)(4\beta(j-1)b+a^2)^3)$, which possesses the same positive or negative to $\eta_{15} = 64b^3\delta(j-1)^3\beta^3 + (48((j-1)^2\delta-(2/3)j))b^2a^2\beta^2 + (12((j-1)\delta + (2/3)j + 1/3))ba^4\beta + a^6(\delta-1)$. Recall that $a^2 < \frac{1}{2}b\beta$, we can prove that if $\delta < \delta' = 8\frac{a^2}{b\beta}(4-\frac{a^2}{b\beta})(-(1/8)\frac{a^2}{b\beta}+j)/(4j+\frac{a^2}{b\beta}-4)^3$ and $j < \frac{a^2}{8b\beta}$, a higher backward shareholding rate j can provide higher profits to the retailer, otherwise the retailer prefers a pure forward shareholding strategy. Moreover, this result remains in the cross-shareholding strategy.

Appendix D. Further discussion of agency selling with shareholding

D.1 The monotonicity of the optimal solutions under agency selling

Under agency selling, the optimal green level is $\theta^{\mathrm{A}*} = \frac{(-Ax-bc+A)a}{a^2x-a^2+2b\beta}$. Therefore, it has $\frac{\partial \theta^{\mathrm{A}*}}{\partial x} = -\frac{ab(A\beta-2a^2c)}{(2b\beta+(x-1)^2)^2}$. Notice that $(A\beta - 2a^2c) > 0$, $\frac{\partial \theta^{\mathrm{A}*}}{\partial x} < 0$ is always valid.

For the optimal retail price, we have $p^{\mathrm{A}*} = \frac{-a^2cx + A\beta x + a^2c - b\beta c - A\beta}{a^2x^2 - 2a^2x + 2b\beta x + a^2 - 2b\beta}$ and $\frac{\partial p^{\mathrm{A}*}}{\partial x} = \frac{c(x-1)^2a^4 - \beta(x-1)(Ax-2bc-A)a^2 + 2b^2c\beta^2}{(x-1)^2((x-1)a^2+2b\beta)^2}$. While the denominator of $\frac{\partial p^{\mathrm{A}*}}{\partial x}$ is definitely positive, we focus on the numerator of it. Let $\eta_{13} = c(x-1)^2a^4 - \beta(x-1)(Ax-2bc-A)a^2 + 2b^2c\beta^2$, we can easily obtain that $\frac{\partial^2 \eta_{13}}{\partial x^2} = 2a^4c - 2Aa^2\beta$. When $x = 0$, $\frac{\partial \eta_{13}}{\partial x} = (2((bc+A)\beta - ca^2))a^2$, then we get η_{13} increasing to the sharing ratio x in the interval of (0, 1). Furthermore, solve $\eta_{13} = 0$, we have $x_7 = \frac{1}{(-a^2c+A\beta)a}(-ca^3 + ab\beta c + Aa\beta \pm \sqrt{-a^2b^2\beta^2c^2 + 2Ab^2\beta^3c})$, while the larger root is omitted. Then if $x < x_7$, $\frac{\partial p^{\mathrm{A}*}}{\partial x} < 0$, otherwise $\frac{\partial p^{\mathrm{A}*}}{\partial x} > 0$.

D.2 The comparison of optimal solutions under all schemes and the corresponding strategies

(i) Comparing the optimal green level under all the schemes and the corresponding strategies, it follows

$$\theta^{\mathrm{A}*} - \theta^{\mathrm{R}*} = \frac{ab(4Ax+2bc-2A)\beta - a^3bcx}{(a^2-4b\beta)(2b\beta+a^2(x-1))}.$$

We can easily obtain that $\theta^{\mathrm{A}*} - \theta^{\mathrm{R}*}$ possess the same monotonicity to θ_{A} and is always decreasing to the agency sharing ratio x. Solve $\theta^{\mathrm{A}*} - \theta^{\mathrm{R}*} = 0$, we get $x = \frac{2\beta(A-bc)}{4A\beta - a^2c}$. Therefore, if

$x < x_{\theta 1} = \dfrac{2\beta(A-bc)}{4A\beta - a^2c}$, $\theta^{\mathrm{A}*} > \theta^{\mathrm{R}*}$, otherwise $\theta^{\mathrm{A}*} < \theta^{\mathrm{R}*}$. Similarly, comparing with the optimal green level under agency selling with shareholdings, we get the following results.
If $x < x_{\theta 2} = \dfrac{2\beta(k-1)(-bc+A)}{2(k-2)\beta A + a^2c}$, $\theta^{\mathrm{A}*} > \theta^{\mathrm{SF}*}$, otherwise $\theta^{\mathrm{A}*} < \theta^{\mathrm{SF}*}$;
if $x < x_{\theta 3} = \dfrac{2\beta(2j-1)(-bc+A)}{4(j-1)\beta A + a^2c}$, $\theta^{\mathrm{A}*} > \theta^{\mathrm{SB}*}$, otherwise $\theta^{\mathrm{A}*} < \theta^{\mathrm{SB}*}$;
if $x < x_{\theta 4} = \dfrac{2\beta((2-k)j+k-1)(-bc+A)}{2\beta(k-2)(1-j)\beta A + a^2c}$, $\theta^{\mathrm{A}*} > \theta^{\mathrm{SC}*}$, otherwise $\theta^{\mathrm{A}*} < \theta^{\mathrm{SC}*}$.
Comparing the above thresholds, we can discuss $\dfrac{1}{2}$, $\dfrac{1-k}{2-k}$, $\dfrac{1-2j}{2-2j}$ and $\dfrac{(k-2)(j-1)-1}{(k-2)(j-1)}$, which are all in forms of $\Psi = \dfrac{\psi - 1}{\psi}$. While $\dfrac{\partial \Psi}{\partial \psi} = \dfrac{1}{\psi^2} > 0$, it only has to analyze 2, $(2-k)$, $(2-2j)$ and $(2-k)(1-j)$. We observe that x_{θ_1} is the largest and x_{θ_4} is the smallest. For the comparison of x_{θ_2} and x_{θ_3}, if $k < 2j$, $x_{\theta_2} > x_{\theta_3}$, otherwise $x_{\theta_2} < x_{\theta_3}$. Combining with the results in Proposition 1, the relationship between the optimal green levels under all the schemes (or strategies) can be obtained.
(ii) Similarly, for the optimal retail price, we have

$$x_{p1} = \frac{1}{2(a^2(-a^2c + b\beta c + 3A\beta))}\Big(-a^4c - 2b^2\beta^2 c + 5Aa^2\beta - 2Ab\beta^2 + \sqrt{4b^2(bc+A)^2\beta^4 + a^8c^2 + 4a^2b(-2b^2c^2 - 9Abc + A^2)\beta^3 + a^4(10bc+A)(2bc+A)\beta^2 - 2a^6c(4bc+A)\beta}\Big),$$

if $x < x_{p1}$, $p^{\mathrm{A}*} < p^{\mathrm{R}*}$, otherwise $p^{\mathrm{A}*} > p^{\mathrm{R}*}$. For the comparisons of the optimal retail price under agency selling with that under different shareholding strategies, it has

$$x_{p2} = \frac{1}{4((-(1/2)bc + A(k-3/2))\beta + (1/2)a^2c)a^2}\Big(((-2k+2)bA + 2b^2c)\beta^2 + a^4c - 2a^2bck\beta + (4k-5)a^2A\beta - \sqrt{(-8a^2b^3\beta^3 + 4b^4\beta^4 + 4a^4\beta^2(k^2-4k+5)b^2 + 4a^6\beta(k-2)b + a^8)c^2 + 2a^6\beta Ac - (2(4\beta^3(k-1)b^3 + (4(k^2-5k+9/2))\beta^2a^2b^2 + (4k-6)\beta a^4b))\beta Ac + (-2\beta(k-1)b + a^2)^2\beta^2A^2}\Big),$$

$$x_{p3} = \frac{1}{6(a^2((j-1)((1/3)bc + A)\beta + (1/3)a^2c))}\Big(((-2j+2)cb^2 + (-2j+2)bA)\beta^2 + (-2a^2bcj + (6j-5)a^2A)\beta + a^4c - \sqrt{4b^2(j-1)^2(bc+A)^2\beta^4 - (4(-2b^2c^2 + 8c(j-9/8)Ab + A^2))a^2(j-1)b\beta^3 - 16c(j-3/4)Ab + A^2)\beta^2 - 2ca^6(-4c(j-1)b + A)\beta + a^8c^2 + a^4(16c^2(j^2-2j+5/4)b^2}\Big),$$

and

$$x_{p4} = \frac{1}{a^2((j-1)(-2bc + A(k-6))\beta - 2a^2c)}\Big((((2-2k)j + 2k - 2)bA + (2cj - 2c)b^2)\beta^2 - a^4c + (((4k-6)j - 4k + 5)a^2A + ((2-2k)cj + 2ck)a^2b)\beta + \sqrt{4((k-1)A - bc)^2(j-1)^2b^2\beta^4 + a^8c^2 + 4a^2(j-1)(2b^2c^2 - 2c((k-2)^2j - k^2 + 5k - 9/2)Ab + A^2(k-1))b\beta^3 + a^4(4c^2((k-2)^2j^2 - 2(k-2)^2j + k^2 - 4k + 5)b^2 + 8c((k-2)j - k + 3/2)Ab + A^2)\beta^2 - 2ca^6(2c(k-2)(j-1)b + A)\beta}\Big).$$

Similarly, if $x < x_{p2}$, $p^{\mathrm{A}*} < p^{\mathrm{SF}*}$, otherwise $p^{\mathrm{A}*} > p^{\mathrm{SF}*}$; if $x < x_{p2}$, $p^{\mathrm{A}*} < p^{\mathrm{SB}*}$, otherwise $p^{\mathrm{A}*} > p^{\mathrm{SB}*}$; if $x < x_{p2}$, $p^{\mathrm{A}*} < p^{\mathrm{SC}*}$, otherwise $p^{\mathrm{A}*} > p^{\mathrm{SC}*}$. While the above thresholds are too complex to compare, we only present an numerical example here and all the parameters remain the same to the figures in subsection 7.1.

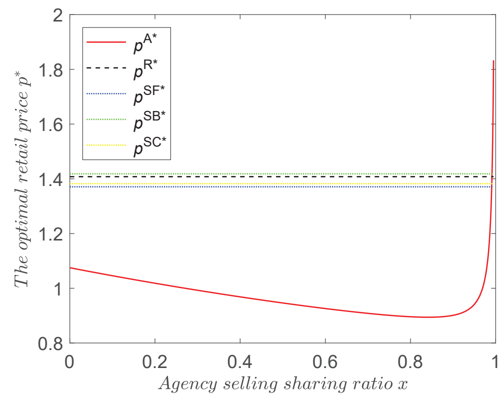

Figure D.12: The comparison of optimal retailer price under different level of agency selling sharing ratio x

Appendix E. Power three green investment cost

Solving the Stackelberg game between the supplier and the retailer, the optimal green level, wholesale price and retail price in the traditional reselling scheme can be obtained as follows.

$$\begin{cases} \theta^* = \frac{1}{12}\sqrt{-\frac{24ab^2\beta c - 24Aab\beta - 2a^4 - 2a\sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6}}{b^2\beta^2}} \\ w^* = \frac{1}{24}\frac{12b^2\beta c + 12Ab\beta + a^3 + \sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6}}{b^2\beta} \end{cases} \tag{E.1}$$

or

$$\begin{cases} \theta^* = \frac{1}{12}\sqrt{-\frac{24ab^2\beta c - 24Aab\beta - 2a^4 + 2a\sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6}}{b^2\beta^2}} \\ w^* = \frac{1}{24}\frac{12b^2\beta c + 12Ab\beta + a^3 - \sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6}}{b^2\beta}, \end{cases} \tag{E.2}$$

while the latter leads to lower profits and can be omitted. Then the corresponding profits of both players are

$$\begin{cases} \pi_s = \frac{1}{3456b^3\beta^2}\Big(-2\beta^3\sqrt{2}\big(\frac{a(-12b^2\beta c + 12Ab\beta + a^3 + \sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6})}{b^2\beta^2}\big)^{3/2}b^3 + \\ 72\beta ba((1/12)\sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6} + (1/12)a^3 + 432b^2(-bc + A)^2\beta^2 - 72a^3b(A - bc)\beta \\ -6a^6 + b\beta(-bc + A))\sqrt{2}\sqrt{\frac{a(-12b^2\beta c + 12Ab\beta + a^3 + \sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6})}{b^2\beta^2}} - \\ 6\sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6}a^3\Big), \\ \pi_r = \frac{1}{2304b^3\beta^2}\Big(2b\beta\Big(\sqrt{\frac{a(-12b^2\beta c + 12Ab\beta + a^3 + \sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6})}{b^2\beta^2}}\sqrt{2}a - 6bc \\ +6A\Big) - a^3 - \sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6}\Big). \end{cases} \tag{E.3}$$

The optimal solutions and the corresponding profits derived from the power three investment cost are displayed as follows.

(i) Under forward shareholding, the supplier and retailer's optimal green level, wholesale price and retail price are displayed as follows.

$$
\begin{cases}
\theta^{\mathrm{SF}*} = -\frac{1}{6}\frac{-\sqrt{-(12(-(1/12)a^3 + b\beta(k-2)(-bc+A)))(k-1)^2a^3} + a^3(k-1)}{(k-2)b\beta a(k-1)}, \\
w^{\mathrm{SF}*} = \frac{1}{6}\frac{1}{b^2\beta(k-2)^2}\Big(2\sqrt{-(3(-(1/12)a^3 + b\beta(k-2)(-bc+A)))(k-1)^2a^3} + ((12c-6ck)b^2 \\
+(6Ak^2 - 18Ak + 12A)b)\beta - a^3k + a^3\Big), \\
p^{\mathrm{SF}*} = \frac{1}{48b^2\beta}\Big(2b(\sqrt{a(-12b^2\beta c + 12Ab\beta + a^3 + \sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6})/(b^2\beta^2)}\sqrt{2}a+ \\
6bc + 18A + \sqrt{-24a^3b^2\beta c + 24Aa^3b\beta + a^6}+)\beta + a^3\Big),
\end{cases}
$$

while the profits of them are

$$
\begin{cases}
\pi_s^{\mathrm{SF}*} = -\frac{1}{54(k-1)^3(k-2)^3b^3\beta^2a^3}\Big((1/4)(k-1)^3a^9 - (9/2)b\beta(k-2)(k-1)^3(-bc+A)a^6+ \\
(27/2)b^2\beta^2(k-2)^2(k-1)^3(-bc+A)^2a^3 + (-(-(1/4)a^3 + 3b\beta(k-2)(A-bc))(k-1)^2a^3)^{3/2} \\
+9(k-1)^2(-(1/12)a^3 + b\beta(k-2)(A-bc))a^3\sqrt{3((1/12)a^3 - b\beta(k-2)(A-bc))(k-1)^2a^3}\Big), \\
\pi_r^{\mathrm{SF}*} = \frac{1}{144(1-k)b^3\beta^2(k-2)^4}\Big(2\sqrt{-(3(-(1/12)a^3 + b\beta(k-2)(-bc+A)))(k-1)^2a^3} - a^3k+ \\
a^3 - 6c\beta(k-1)(k-2)b^2 + 6A\beta(k-1)(k-2)b\Big)^2.
\end{cases}
$$

(ii) Under backward shareholding,

$$
\begin{cases}
\theta^{\mathrm{SB}*} = \frac{1}{6b\beta(1-j)}\sqrt{-a\Big(-\sqrt{-(-(1/4)a^3 + 6b\beta(j-1)(-bc+A))a^3} + 6b\beta(j-1)(-bc+A) \\
-1/2a^3\Big)}, \\
w^{\mathrm{SB}*} = \frac{1}{24b^2\beta(j-1)^2}\Big(2\sqrt{-6a^3(-(1/24)a^3 + b\beta(j-1)(-bc+A))} + ((24cj^2 - 36cj + 12c)b^2 \\
+(-12Aj + 12A)b)\beta + a^3\Big),
\end{cases}
$$

while the profits of them are

$$\begin{cases} \pi_s^{\mathrm{SB}*} = \dfrac{1}{1728(j-1)^2b^3\beta^2}\Big(4\beta^3b^3(j-1)^3(-6a(-(1/6)\sqrt{-a^3(-(1/4)a^3+6b\beta(j-1)(A-bc))}+ b\beta(j-1)(-bc+A)-(1/12)a^3)/(b^2\beta^2(j-1)^2))^{3/2}+72b\beta(j-1)a(-(1/6)\sqrt{-6a^3(-(1/24)a^3+b\beta(j-1)(A-bc))}-6\sqrt{a^3((1/4)a^3-6b\beta(j-1)(-bc+A))}a^3+216b^2(j-1)^2(A-bc)^2\beta^2 -(1/12)a^3)\sqrt{-6a(-(1/6)\sqrt{-6a^3(-(1/24)a^3+b\beta(j-1)(-bc+A))}+b\beta(j-1)(-bc+A)-(1/12)a^3)/(b^2\beta^2(j-1)^2)}+36a^3b(j-1)(-bc+A)\beta-3a^6+b\beta(j-1)(-bc+A)\Big), \\ \pi_r^{\mathrm{SB}*} = \dfrac{-8}{3456(j-1)^2b^3\beta^2}\Big((a(\sqrt{-6a^3(-(1/24)a^3-6b\beta(j-1)(A-bc))}-6b\beta(j-1)(A-1)+ (1/2)a^3)/(b^2\beta^2(j-1)^2)\Big)^{3/2}b^3\beta^3j(j-1)^2+b\Big(24\sqrt{-6a^3(-(1/24)a^3+b\beta(j-1)(-bc+A))}+ 144b\beta(j-1)(-bc+A)+12a^3\Big)\beta(j-1)a\sqrt{a(\sqrt{a^3(1/4a^3-6b\beta(j-1)(-bc+A))}+(1/2)a^3 -6b\beta(j-1)(-bc+A))/(b^2\beta^2(j-1)^2)}+(72b\beta(j-1)(-bc+A)+30a^3)\sqrt{-6a^3(-(1/24)a^3 +b\beta(j-1)(A-bc))}+216b^2(j-1)^2(A-bc)^2\beta^2-144a^3b(j-1)(A-bc)\beta+15a^6\Big). \end{cases}$$

(iii) Under cross-shareholding,

$$\begin{cases} \theta^{\mathrm{SC}*} = \dfrac{1}{6b\beta(k-2)(j-1)}\Big(\sqrt{12}\sqrt{((1/12)a^3+b\beta(k-2)(j-1)(-bc+A)a)}+a^2\Big), \\ w^{\mathrm{SC}*} = \dfrac{1}{6b^2\beta(k-2)^2(j-1)(j+k-1)}\Big(6\beta c((k-2)j-k+1)(k-2)(j-1)b^2+a(k-1)^2 \\ \quad +6A\beta(k-2)(k-1)^2(j-1)b+a(k-1)^2(\sqrt{12}\sqrt{((1/12)a^3+b\beta(k-2)(j-1)(-bc+A))a})\Big), \end{cases}$$

while the profits of them are

$$\begin{cases} \pi_s^{\mathrm{SC}*} = \dfrac{1}{216(k-2)^3b^3(j-1)^2\beta^2}\Big((-(36(k-2))(j-1)ba(-bc+A)\sqrt{(-c\beta(k-2)(j-1)b^2+ A\beta(k-2)(j-1)b+(1/12)a^3)a}\beta+12((((1/12)a^3+b\beta(k-2)(j-1)(-bc+A))a)^{3/2})\sqrt{3}- \\ \quad 3a^4\sqrt{-ac\beta(k-2)(j-1)b^2+Aa\beta(k-2)(j-1)b+(1/12)a^4}-18a^3b(k-2)(j-1)(A-bc)\beta \\ \quad -54b^2(k-2)^2(j-1)^2(-bc+A)^2\beta^2-a^6\Big), \\ \pi_r^{\mathrm{SC}*} = \dfrac{1}{18b^3\beta^2(k-2)^4(1-j)^3}\Big((3(-\beta c(k-1)(k-2)(j-1)^2b^2+A\beta(k-1)(k-2)(j-1)^2b \\ \quad +(1/4((j-2/3)k-(4/3)j+2/3))a^3))\sqrt{3}a\sqrt{-ca\beta(k-2)(-1+j)b^2+Aa\beta(k-2)(-1+j)b+ (1/12)a^4}+(((1/12)a^3+b\beta(k-2)(j-1)(-bc+A))a)^{3/2}j(k-2)\sqrt{3}+(9/2)\beta^2c^2(k-1)(k-2)^2 \\ \quad (j-1)^3b^4-9A\beta^2c(k-1)(k-2)^2(j-1)^3b^3+(9/2(k-2))(A^2\beta(j-1)^2k^2+(-3A^2\beta j^2-a^3cj+ \\ \quad 6A^2\beta j+(2/3)a^3c-3A^2\beta)k+2A^2\beta j^2+((4/3)a^3c-4A^2\beta)j-(2/3)a^3c+2A^2\beta)(j-1)\beta b^2+ \\ \quad (9/2((j-2/3)k-(4/3)j+2/3))(k-2)a^3A(j-1)\beta b+(5/12)a^6((j-3/5)k-(7/5)j+3/5)\Big). \end{cases}$$