On root subsystems and involutions in S_n

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Computational Results

associated with the paper

On root subsystems and involutions in S_n

by

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In [3], we established the following theorem

Theorem 1. *Let t be a standard (λ, μ)-tableau for which* $\mathrm{rev}(t)$ *is column-standard. Let $d \in W$ be the element defined by $t^{\lambda,\mu} d = t$ and let $z = d^{-1} w_{J(\lambda)} d$. Then* $\mathrm{sh}\, z = \lambda'$, *$z \sim_{LR} w_{J(\lambda)}$, and $l(z) = 2l(d) + l(w_{J(\lambda)})$.*

We also determined, using programs in GAP and C, the involutions which are not accounted for by this theorem for some values of n. The partitions corresponding to cells which contain such involutions for the cases $n \leq 12$ are

$n = 9$: 6.3, 3^2.2.1.

$n = 10$: 7.3, 6.3.1, 4.3.2.1, $3^2.2.1^2$.

$n = 11$: 8.3, 7.4, 7.3.1, 6.3.2, $6.3.1^2$, 5.3.2.1, 4^2.2.1, $4.3.2^2$, $4.3.2.1^2$, $3^2.2^2.1$, $3^2.2.1^3$.

$n = 12$: 9.3, 8.4, 8.3.1, 7.4.1, 7.3.2, $7.3.1^2$, 6.3^2, 6.3.2.1, $6.3.1^3$, 5.4.2.1, $5.3.2.1^2$, $4^2.2^2$, $4^2.2.1^2$, $4.3.2^2.1$, $4.3.2.1^3$, 3^3.2.1, $3^2.2^2.1^2$, $3^2.2.1^4$.

We also determined the involutions z with $\mathrm{sh}\, z = \lambda'$, which cannot be written in the form $z = d^{-1} w_{J(\lambda)} d$ for some composition λ with $l(z) = 2l(d) + l(w_{J(\lambda)})$. For each n, the total number of such involutions is denoted by $N_{v,n}$. An investigation carried out in [2] had already shown that $N_{v,n} = 0$ whenever $n \leq 7$.

In the following table, we list the total number $N_{t,n}$ of involutions for $9 \leq n \leq 12$, the number of involutions $N_{b,n}$ not accounted for by the theorem and the fraction $N_{b,n}/N_{t,n}$, together with $N_{v,n}$ and $N_{v,n}/N_{t,n}$.

n	$N_{t,n}$	$N_{b,n}$	$N_{b,n}/N_{t,n}$	$N_{v,n}$	$N_{v,n}/N_{t,n}$
9	2620	12	0.00458	4	0.00153
10	9496	58	0.00611	22	0.00232
11	35696	418	0.01171	142	0.00398
12	140152	2234	0.01594	870	0.00621

The following tables give more detailed information. In these tables, the column entries give (i) the partition λ; (ii) the number $N_{t,\lambda}$ of standard tableaux whose shape is the partition λ, and this is also the number of involutions in the corresponding two-sided cell; (iii) the number $N_{b,\lambda}$ of involutions in the two-sided cell corresponding to λ which are not accounted for by the theorem.

$n = 9$

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
9	1	0
8.1	8	0
7.2	27	0
7.1^2	28	0
6.3	48	4
6.2.1	105	0
6.1^3	56	0
5.4	42	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
5.3.1	162	0
5.2^2	120	0
$5.2.1^2$	189	0
5.1^4	70	0
$4^2.1$	84	0
4.3.2	168	0
$4.3.1^2$	216	0
$4.2^2.1$	216	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
$4.2.1^3$	189	0
4.1^5	56	0
3^3	42	0
$3^2.2.1$	168	8
$3^2.1^3$	120	0
3.2^3	84	0
$3.2^2.1^2$	162	0
$3.2.1^4$	105	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
3.1^6	28	0
$2^4.1$	42	0
$2^3.1^3$	48	0
$2^2.1^5$	27	0
2.1^7	8	0
1^9	1	0

$n = 10$

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
10	1	0
9.1	9	0
8.2	35	0
8.1^2	36	0
7.3	75	4
7.2.1	160	0
7.1^3	84	0
6.4	90	0
6.3.1	315	18
6.2^2	225	0
$6.2.1^2$	350	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
6.1^4	126	0
5^2	42	0
5.4.1	288	0
5.3.2	450	0
$5.3.1^2$	567	0
$5.2^2.1$	525	0
$5.2.1^3$	448	0
5.1^5	126	0
$4^2.2$	252	0
$4^2.1^2$	300	0
4.3^2	210	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
4.3.2.1	768	10
$4.3.1^3$	525	0
4.2^3	300	0
$4.2^2.1^2$	567	0
$4.2.1^4$	350	0
4.1^6	84	0
$3^3.1$	210	0
$3^2.2^2$	252	0
$3^2.2.1^2$	450	26
$3^2.1^4$	225	0
$3.2^3.1$	288	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
$3.2^2.1^3$	315	0
$3.2.1^5$	160	0
3.1^7	36	0
2^5	42	0
$2^4.1^2$	90	0
$2^3.1^4$	75	0
$2^2.1^6$	35	0
2.1^8	9	0
1^{10}	1	0

$n = 11$

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
1^2	1	0
10.1	10	0
9.2	44	0
9.1^2	45	0
8.3	110	14
8.2.1	231	0
8.1^3	120	0
7.4	165	10
7.3.1	550	18
7.2^2	385	0
$7.2.1^2$	594	0
7.1^4	210	0
6.5	132	0
6.4.1	693	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
6.3.2	990	50
$6.3.1^2$	1232	50
$6.2^2.1$	1100	0
$6.2.1^3$	924	0
6.1^5	252	0
$5^2.1$	330	0
5.4.2	990	0
$5.4.1^2$	1155	0
5.3^2	660	0
5.3.2.1	2310	30
$5.3.1^3$	1540	0
5.2^3	825	0
$5.2^2.1^2$	1540	0
$5.2.1^4$	924	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
5.1^6	210	0
$4^2.3$	462	0
$4^2.2.1$	1320	38
$4^2.1^3$	825	0
$4.3^2.1$	1188	0
$4.3.2^2$	1320	50
$4.3.2.1^2$	2310	32
$4.3.1^4$	1100	0
$4.2^3.1$	1155	0
$4.2^2.1^3$	1232	0
$4.2.1^5$	594	0
4.1^7	120	0
$3^3.2$	462	0
$3^3.1^2$	660	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
$3^2.2^2.1$	990	70
$3^2.2.1^3$	990	56
$3^2.1^5$	385	0
3.2^4	330	0
$3.2^3.1^2$	693	0
$3.2^2.1^4$	550	0
$3.2.1^6$	231	0
3.1^8	45	0
$2^5.1$	132	0
$2^4.1^3$	165	0
$2^3.1^5$	110	0
$2^2.1^7$	44	0
2.1^9	10	0
1^{11}	1	0

$n = 12$

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
12	1	0
11.1	11	0
10.2	54	0
10.1^2	55	0
9.3	154	14
9.2.1	320	0
9.1^3	165	0
8.4	275	14
8.3.1	891	78
8.2^2	616	0
$8.2.1^2$	945	0
8.1^4	330	0
7.5	297	0
7.4.1	1408	92
7.3.2	1925	54
$7.3.1^2$	2376	50
$7.2^2.1$	2079	0
$7.2.1^3$	1728	0
7.1^5	462	0
6^2	132	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
6.5.1	1155	0
6.4.2	2673	0
$6.4.1^2$	3080	0
6.3^2	1650	236
6.3.2.1	5632	256
$6.3.1^3$	3696	110
6.2^3	1925	0
$6.2^2.1^2$	3564	0
$6.2.1^4$	2100	0
6.1^6	462	0
$5^2.2$	1320	0
$5^2.1^2$	1485	0
5.4.3	2112	0
5.4.2.1	5775	80
$5.4.1^3$	3520	0
$5.3^2.1$	4158	0
$5.3.2^2$	4455	0
$5.3.2.1^2$	7700	122
$5.3.1^4$	3564	0
$5.2^3.1$	3520	0

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
$5.2^2.1^3$	3696	0
$5.2.1^5$	1728	0
5.1^7	330	0
4^3	462	0
$4^2.3.1$	2970	0
$4^2.2^2$	2640	88
$4^2.2.1^2$	4455	128
$4^2.1^4$	1925	0
$4.3^2.2$	2970	0
$4.3^2.1^2$	4158	0
$4.3.2^2.1$	5775	292
$4.3.2.1^3$	5632	68
$4.3.1^5$	2079	0
4.2^4	1485	0
$4.2^3.1^2$	3080	0
$4.2^2.1^4$	2376	0
$4.2.1^6$	945	0
4.1^8	165	0
3^4	462	0
$3^3.2.1$	2112	204

λ	$N_{t,\lambda}$	$N_{b,\lambda}$
$3^3.1^3$	1650	0
$3^2.2^3$	1320	0
$3^2.2^2.1^2$	2673	248
$3^2.2.1^4$	1925	100
$3^2.1^6$	616	0
$3.2^4.1$	1155	0
$3.2^3.1^3$	1408	0
$3.2^2.1^5$	891	0
$3.2.1^7$	320	0
3.1^9	55	0
2^6	132	0
$2^5.1^2$	297	0
$2^4.1^4$	275	0
$2^3.1^6$	154	0
$2^2.1^8$	54	0
2.1^{10}	11	0
1^{12}	1	0

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