SUBROUTINES USED IN ALL MODELS

CALKA FUNCTION

```
function cc=calka(x,w,w0,a0,a1,a2)

r=20;
M=length(x)-r;
ca=zeros(1,length(x)-1);

Y=zeros(M,1);
for ii=1:M
Y(ii,1)=w(ii);
end

C=zeros(M,M);
C(1,1)=(x(2)-x(1))*((x(3)-x(2))^2);
C(1,2)=(x(2)-x(1))*(x(3)-x(2))*(x(3)-2*x(2)+x(1));
C(1,3)=(x(2)-x(3))*((x(2)-x(1))^2);
C(end,end)=1;
for ii=1:M-2
C(ii+1,ii)=x(ii+2)-x(ii+1);
C(ii+1,ii+1)=2*(x(ii+2)-x(ii));
C(ii+1,ii+2)=x(ii+1)-x(ii);
end

B=zeros(M,M);
B(1,1)=-2/3*((x(3)-x(2))^2);
B(1,2)=2/3*(((x(3)-x(2))^2)+((x(2)-x(1))^2));
B(1,3)=-2/3*((x(2)-x(1))^2);
for ii=1:M-2
B(ii+1,ii)=-(x(ii+2)-x(ii+1))/(x(ii+1)-x(ii));
B(ii+1,ii+1)=((x(ii+2)-x(ii+1))/(x(ii+1)-x(ii)))-((x(ii+1)-x(ii))/(x(ii+2)-
x(ii+1)));
B(ii+1,ii+2)=(x(ii+1)-x(ii))/(x(ii+2)-x(ii+1));
end

G=zeros(M,1);
zz1=(1-x(M+1))^a2/(((1-x(M))^a1)*((1-x(M+1))^a2)-((1-x(M))^a2)*((1-
x(M+1))^a1))*(w(M)-w0*(1-x(M))^a0)-(1-x(M))^a2/(((1-x(M))^a1)*((1-
x(M+1))^a2)-((1-x(M))^a2)*((1-x(M+1))^a1))*(w(M+1)-w0*(1-x(M+1))^a0);
zz2=-((1-x(M+1))^a1)/(((1-x(M))^a1)*((1-x(M+1))^a2)-((1-x(M))^a2)*((1-
x(M+1))^a1))*(w(M)-w0*(1-x(M))^a0)+(1-x(M))^a1/(((1-x(M))^a1)*((1-
x(M+1))^a2)-((1-x(M))^a2)*((1-x(M+1))^a1))*(w(M+1)-w0*(1-x(M+1))^a0);
G(end,1)=-w0*a0*(1-x(M))^(a0-1)-zz1*a1*(1-x(M))^(a1-1)-zz2*a2*(1-x(M))^(a2-
1);

D=zeros(M-1,M);
for ii=1:M-1
D(ii,ii)=(x(ii+1)-x(ii))^2;
D(ii,ii+1)=-((x(ii+1)-x(ii))^2);
end

F=zeros(M-1,M);
for ii=1:M-1
F(ii,ii)=x(ii+1)-x(ii);
F(ii,ii+1)=x(ii+1)-x(ii);
end

S=1/4*D*inv(C)*B+1/2*F;
L=1/12*D*inv(C)*G;
```

```
z1=S*Y+L;

for ii=M:length(x)-1
y1=1-x(ii);
y2=1-x(ii+1);
w1=w(ii);
w2=w(ii+1);
wz=y2^a1*y1^a2-y2^a2*y1^a1;
a11=-y2^a2/wz;
a12=y1^a2/wz;
a13=(y1^a0*y2^a2-y2^a0*y1^a2)/wz;
a21=y2^a1/wz;
a22=-y1^a1/wz;
a23=(y2^a0*y1^a1-y1^a0*y2^a1)/wz;
z2(ii-M+1)=w0*(1/(1+a0)*(y1^(1+a0)-y2^(1+a0))+a13*(y1^(1+a1)-
2^(1+a1))/(1+a1)+a23*(y1^(1+a2)-y2^(1+a2))/(1+a2))+w1*(a11*(y1^(1+a1)-
y2^(1+a1))/(1+a1)+a21*(y1^(1+a2)-y2^(1+a2))/(1+a2))+w2*(a12*(y1^(1+a1)-
y2^(1+a1))/(1+a1)+a22*(y1^(1+a2)-y2^(1+a2))/(1+a2));
clear y1 y2 w1 w2 wz
end

ca(1,1:M-1)=z1;
ca(1,M:end)=z2;

end
```

NEWTON-RAPHSON FUNCTION

```
function w=f_z(a_i,alf_i,c,x0)

x_N=x0;
ff=sum(a_i.*x_N.^alf_i)+c;
ffp=sum(alf_i.*a_i.*x_N.^(alf_i-1));
x_N1=x_N-ff/ffp;
while abs((x_N-x_N1)/x_N1)>=1e-15
x_N=x_N1;
ff=sum(a_i.*x_N.^alf_i)+c;
ffp=sum(alf_i.*a_i.*x_N.^(alf_i-1));
x_N1=x_N-ff/ffp;
end

w=x_N1;

end
```

MESH FUNCTION

```
function ww=sgl(N,delta)

alpha=3;
beta=1.2;
eps=delta;
M=10*N+1;
x=0:1/(M-1):1;
xx=0:1/(N-1):1;
xx=(1-eps^(1/alpha))*xx;

mm=floor(M/2)+1;

AAA=(1-(x(mm))^alpha)/(x(mm))^beta;
```

```
for ii=1:M
if ii<M/2
w(ii)=AAA*x(ii)^beta;
else
w(ii)=1-(1-x(ii))^alpha;
end
end

for ii=1:M
if ii<M/2
w1(ii)=beta*AAA*x(ii)^(beta-1);
else
w1(ii)=alpha*(1-x(ii))^(alpha-1);
end
end

r2=round(M/5);
r3=round(2*M/3);

in=0;
for ii=1:M
if x(ii)<x(r2)
in=in+1;
xe(in)=x(ii);
we(in)=AAA*x(ii)^beta;
elseif x(ii)>x(r3)
in=in+1;
xe(in)=x(ii);
we(in)=1-(1-x(ii))^alpha;
end
end

y=spaps(xe,we,0,3);

for ii=1:length(xx)
if xx(ii)==0
wg(ii)=0;
elseif ii==length(xx)
wg(ii)=1-delta;
else
wg(ii)=fnval(xx(ii),y);
end
end

ww=wg;

end
```

CALKA SPLINE FUNCTION

```
function cc=calka_spline(x,w)

ak=spline(x,[0 w 0]);
cc=fnval(1,fnint(ak))-fnval(0,fnint(ak));

end
```

PKN SELF SIMILAR

MAIN SCRIPT

```
clear all
clc
format long

N=200;
eps=1e-7;
x=sgl(N,eps);

n=1;
wskaznik=1e-10;

alfa_0=1/(n+2);
alfa_1=alfa_0+1;
alfa_2=alfa_1+1;
alfa_3=alfa_2+1;
b0=1;
b1=b0+1;
b2=b1+1;

CA=alfa_0;
gama=1/(3+2*n);
miu=alfa_0^(1/n)*(1+n)/(n+gama*(2+n));
beta=miu*gama+alfa_0^(1/n);

q0=1;
ql0=0;
ql=zeros(length(x),1);
delta_ql=zeros(length(x),1);
int_ql=0;
int_del_ql=zeros(1,length(x)-1);

hg=(1-x).^alfa_0;
delta_hg=(1-x).^alfa_1;

[h0,f0,fi,del_fi]=blok_phi(x,hg,delta_hg,alfa_0,alfa_1,alfa_2,n,q0,int_ql,i
nt_del_ql,ql0,beta,miu,b0,b1,b2);

[hf,delta_hf]=blok_h(x,delta_hg,hg,h0,del_fi,f0,alfa_0,alfa_1,alfa_2,n);
%[hf,delta_hf]=blok_h_U(x,delta_phi,h0,f0,alfa_0,nn);

wsk=sqrt(trapz(x,(delta_hf-delta_hg).^2)/trapz(x,delta_hf.^2))

while wsk>wskaznik

hh=hf;
delta_hh=delta_hf;
clear h0 f0 fi del_fi hf delta_hf
[h0,f0,fi,del_fi]=blok_phi(x,hh,delta_hh,alfa_0,alfa_1,alfa_2,n,q0,int_ql,i
nt_del_ql,ql0,beta,miu,b0,b1,b2);
[hf,delta_hf]=blok_h(x,delta_hh,hh,h0,del_fi,f0,alfa_0,alfa_1,alfa_2,n);
%[hf,delta_hf]=blok_h_U(x,delta_phi,h0,f0,alfa_0,nn);

wsk=sqrt(trapz(x,(delta_hf-delta_hh).^2)/trapz(x,delta_hf.^2))

end
```

FUNCTION FOR PHI

```
function
[h0_1,f0_1,phi,delta_phi]=blok_phi(x,h,delta_h,alfa_0,alfa_1,alfa_2,nn,q0,i
nt_ql,int_del_ql,ql_0,beta,miu,b0,b1,b2)

int_h=calka(x,h,0,0,alfa_0,alfa_1);
sumf=sum(int_h);

pp1=-miu*int_ql-beta*sumf;
pt1=1+2/nn;
h0_1=f_z(pp1,pt1,q0,10);
f0_1=1/(alfa_0+1)*(miu*ql_0*h0_1^(2/nn)+beta*h0_1^(1+2/nn));

int_del_h=calka(x,delta_h,0,alfa_0,alfa_1,alfa_2);
mh1=miu*int_del_ql+beta*int_del_h;

for jj=1:length(x)-1
mh2(jj)=(1-x(jj+1))*delta_h(jj+1)-(1-x(jj))*delta_h(jj);
end

FF=-h0_1^(1+2/nn)*mh1-f0_1*mh2;

del_1=b1;
del_2=b2;
y=1-x;
wz=y(end-2)^del_1*y(end-1)^del_2-y(end-2)^del_2*y(end-1)^del_1;
K_1=(y(end-1)^del_2*y(end)^del_1-y(end-1)^del_1*y(end)^del_2)/wz;
K_2=(y(end-2)^del_1*y(end)^del_2-y(end-2)^del_2*y(end)^del_1)/wz;

ddf=-(h(end-1)*K_1*(FF(end)+FF(end-1))+K_2*FF(end)*h(end-2))/(h(end-
1)*h(end-2)-K_1*h(end)*h(end-1)-K_2*h(end)*h(end-2));

fdf=ddf;
for ii=1:length(x)-1
fdf(ii+1)=(fdf(ii)*h(end-ii+1)-FF(end-ii+1))/h(end-ii);
end

for ii=1:length(fdf)
fdd(ii)=fdf(end-ii+1);
end

delta_phi=fdd;
phi=delta_phi+f0_1*(1-x).^b0;

end
```

FUNCTION FOR W

```
function
[h,delta_h]=blok_h(x,delta_hg,hg,h0,delta_phi,f0,alfa_0,alfa_1,alfa_2,nn)

dFF=(1-(1+1/h0./(1-x).^alfa_0.*delta_hg).^(nn+1))./(hg.^(nn+1));

As=(f0-alfa_0^(1/nn)*h0^(1+2/nn))/(alfa_0^(1/nn)*h0^(1+2/nn));
etax=(1+As*(1-x)+delta_phi./(alfa_0^(1/nn)*h0^(1+2/nn))).^nn-1-As*nn*(1-x);

for jj=1:length(x)-1
```

```
cf1(1,jj)=1/(2-alfa_0*(nn+1))*((1-x(jj))^(2-alfa_0*(nn+1))-(1-x(jj+1))^(2-
alfa_0*(nn+1)));
end

f2=(1-x).^(-alfa_0*(nn+1)).*etax;
cf2=calka(x,f2,0,-alfa_0*(nn+1)+1,-alfa_0*(nn+1)+2,-alfa_0*(nn+1)+3);

f3=dFF.*((f0-alfa_0^(1/nn)*h0^((nn+2)/nn))*(1-
x)+delta_phi+alfa_0^(1/nn)*h0^((nn+2)/nn)).^nn;
cf3=calka(x,f3,0,-alfa_0*(nn+1),1-alfa_0*(nn+1),2-alfa_0*(nn+1));

FI=-(f0-alfa_0^(1/nn)*h0^(1+2/nn))/(alfa_0^(1/nn-1)*h0^(2/nn))*nn*cf1-
alfa_0*h0*cf2-cf3;

ww=0;
a0=alfa_0;
a1=alfa_1;
a2=alfa_2;

x2=x(end-1);
x4=x(end-2);
x0=x(end);

wz= (1-x2)^a2*(1-x4)^a1-(1-x2)^a1*(1-x4)^a2;
AA1=((1-x0)^a2*(1-x4)^a1-(1-x0)^a1*(1-x4)^a2)/wz;
AA2=((1-x0)^a1*(1-x2)^a2-(1-x0)^a2*(1-x2)^a1)/wz;
AA3=(1-x0)^a0+((1-x0)^a2*(1-x2)^a1*(1-x4)^a0-(1-x0)^a1*(1-x2)^a2*(1-x4)^a0-
(1-x0)^a2*(1-x2)^a0*(1-x4)^a1+(1-x0)^a1*(1-x2)^a0*(1-x4)^a2)/wz;

FF=zeros(length(x),1);
FF(end,1)=AA3*ww;
for jj=1:length(x)-1
FF(jj,1)=FI(jj);
end

MM=zeros(length(x),length(x));

for jj=1:length(x)-1
MM(jj,jj)=-1;
MM(jj,jj+1)=1;
end

MM(end,end-2)=-AA2;
MM(end,end-1)=-AA1;
MM(end,end)=1;

M=inv(MM);

w=M*FF;

delta_h=w';
h=delta_h+h0*(1-x).^alfa_0;

end
```

FUNCTION FOR W (U)

```
function [h,delta_h]=blok_h_U(x,delta_fi,h0,f0,alfa_0,n)

V0=alfa_0^(1/n)*h0^((n+2)/n);
As=f0/V0-1;
```

```
etax=(1+As*(1-x)+delta_fi/V0).^n-1-As*n*(1-x);

for jj=1:length(x)-1
cf1(1,jj)=x(jj+1)-1/2*x(jj+1)^2-x(jj)+1/2*x(jj)^2;
end

cf2=calka(x,etax,0,1,2,3);

FI=-h0^(n+2)*(n*As*cf1+cf2);

ww=0;
a0=1;
a1=2;
a2=3;

x2=x(end-1);
x4=x(end-2);
x0=x(end);

wz= (1-x2)^a2*(1-x4)^a1-(1-x2)^a1*(1-x4)^a2;
AA1=((1-x0)^a2*(1-x4)^a1-(1-x0)^a1*(1-x4)^a2)/wz;
AA2=((1-x0)^a1*(1-x2)^a2-(1-x0)^a2*(1-x2)^a1)/wz;
AA3=(1-x0)^a0+((1-x0)^a2*(1-x2)^a1*(1-x4)^a0-(1-x0)^a1*(1-x2)^a2*(1-x4)^a0-
(1-x0)^a2*(1-x2)^a0*(1-x4)^a1+(1-x0)^a1*(1-x2)^a0*(1-x4)^a2)/wz;

FF=zeros(length(x),1);
FF(end,1)=AA3*ww;
for jj=1:length(x)-1
FF(jj,1)=FI(jj);
end

MM=zeros(length(x),length(x));
for jj=1:length(x)-1
MM(jj,jj)=-1;
MM(jj,jj+1)=1;
end

MM(end,end-2)=-AA2;
MM(end,end-1)=-AA1;
MM(end,end)=1;

M=inv(MM);

delta_u=M*FF;

U=h0^(n+2)*(1-x)+delta_u';
h=U.^(1/(n+2));
delta_h=h-h0*(1-x).^alfa_0;

end
```

PKN TRANSIENT

MAIN SCRIPT

```
clear all
clc
format long
```

```
N=100;
eps=1e-6;
nn=.3;
wskaznik=1e-9;

x=sgl(N,eps);

MM=100;
ll=3*100/MM-2*0.01;
for ii=1:MM
t(ii)=0.01*(ii-1)+(1/3)*(ll-0.01)/((MM-1)^2)*(ii-1)^3;
end
ts=t(1);

aa=1;

alfa_0=1/(nn+2);
alfa_1=alfa_0+1;
alfa_2=alfa_1+1;
alfa_3=alfa_2+1;
b0=1;
b1=b0+1;
b2=b1+1;

CA=alfa_0;
m=1;
gama=1/(3+2*nn);
beta=(((m+2)/nn)*gama+2*gama+1)*CA^(1/nn)/(((nn+2)/nn)*gama+1);

w0m(1)=w0(1);
f0m(1)=f0(1);
Lm(1)=LL(1);
V0m(1)=V0(1);
fim(1,:)=fi(1,:);
delta_fim(1,:)=delta_fi(1,:);
wm(1,:)=w(1,:);
delta_wm(1,:)=delta_w(1,:);
dwdtm(1,:)=dwdt(1,:);
deltawdtm(1,:)=deltawdt(1,:);
dw0dtm(1)=dw0dt(1);

for ii=1:length(t)-1
dt=t(ii+1)-t(ii);
[w0_ost,f0_ost,V0_ost,L_ost,phi_ost,delta_phi_ost,w_ost,delta_w_ost,dwdt_os
t,deltawdt_ost,dw0dt_ost]=blok_czasowy(x,dt,delta_wm(ii,:),deltawdtm(ii,:),
w0m(ii),dw0dtm(ii),delta_fim(ii,:),f0m(ii),alfa_0,alfa_1,alfa_2,b0,b1,b2,Lm
(ii),q0(ii+1),int_ql(ii+1),int_del_ql(ii+1,:),ql0(ii+1),wm(ii,:),dwdtm(ii,:
),nn);
w0m(ii+1)=w0_ost;
f0m(ii+1)=f0_ost;
V0m(ii+1)=V0_ost;
Lm(ii+1)=L_ost;
fim(ii+1,:)=phi_ost;
Vm(ii+1,:)=phi_ost+x*V0_ost;
delta_fim(ii+1,:)=delta_phi_ost;
wm(ii+1,:)=w_ost;
delta_wm(ii+1,:)=delta_w_ost;
dwdtm(ii+1,:)=dwdt_ost;
deltawdtm(ii+1,:)=deltawdt_ost;
dw0dtm(ii+1)=dw0dt_ost;
end
```

BLOK CZASOWY

```
function
[w0_ost,f0_ost,V0_ost,L_ost,phi_ost,delta_phi_ost,w_ost,delta_w_ost,dwdt_os
t,deltawdt_ost,dw0dt_ost]=blok_czasowy(x,dt,delta_w_poprz,deltawdt_poprz,w0
_poprz,dw0dt_poprz,delta_fi_poprz,f0_poprz,alfa_0,alfa_1,alfa_2,b0,b1,b2,L_
poprz,q0,int_ql,int_del_ql,ql0,w_poprz,dwdt_poprz,nn)

r1=2;
r2=r1-1;

delta_ww=delta_w_poprz+deltawdt_poprz*dt;
ww=w_poprz+dwdt_poprz*dt;
ww0=w0_poprz+dw0dt_poprz*dt;
delta_ff=delta_fi_poprz;

[f0m,w0m1,Lm,V0m,delta_fim]=modul_na_phi(x,dt,alfa_0,alfa_1,alfa_2,b1,b2,L_
poprz,w0_poprz,delta_w_poprz,deltawdt_poprz,dw0dt_poprz,delta_ww,ww0,delta_
ff,f0_poprz,q0,int_ql,int_del_ql,ql0,r1,r2,nn);
w0m=w0m1;

fim=f0m*(1-x).^b0+delta_fim;

[wm,delta_wm]=modul_na_w(x,w0m,f0m,V0m,Lm,delta_ww,ww,delta_fim,nn,alfa_0,a
lfa_1,alfa_2);

wsk=sqrt(trapz(x,(delta_wm-delta_ww).^2)/trapz(x,delta_wm.^2));

alk_w=.9;
alk_fi=.6;

while wsk>1e-9
delta_ww=alk_w*delta_ww+(1-alk_w)*delta_wm;
ww=alk_w*ww+(1-alk_w)*wm;
w0m=alk_w*w0m+(1-alk_w)*w0m1;

[f0m1,w0m1,Lm1,V0m1,delta_fim1]=modul_na_phi(x,dt,alfa_0,alfa_1,alfa_2,b1,b
2,L_poprz,w0_poprz,delta_w_poprz,deltawdt_poprz,dw0dt_poprz,delta_ww,w0m,de
lta_fim,f0m,q0,int_ql,int_del_ql,ql0,r1,r2,nn);

f0m=alk_fi*f0m+(1-alk_fi)*f0m1;
delta_fim=alk_fi*delta_fim+(1-alk_fi)*delta_fim1;
fim=f0m*(1-x).^b0+delta_fim;


[wm,delta_wm]=modul_na_w(x,w0m1,f0m,V0m1,Lm1,delta_ww,ww,delta_fim,nn,alfa_
0,alfa_1,alfa_2);

wsk=sqrt(trapz(x,(delta_wm-delta_ww).^2)/trapz(x,delta_wm.^2));

end

w0_ost=w0m1;
f0_ost=f0m;
V0_ost=V0m1;
L_ost=Lm1;
phi_ost=fim;
delta_phi_ost=delta_fim;
delta_w_ost=delta_wm;
```

```
w_ost=wm;
dwdt_ost=r1/dt*(w_ost-w_poprz)-r2*dwdt_poprz;
deltawdt_ost=r1/dt*(delta_ww-delta_w_poprz)-r2*deltawdt_poprz;
dw0dt_ost=r1/dt*(w0_ost-w0_poprz)-r2*dw0dt_poprz;

end
```

FUNCTION FOR PHI

```
function
[f0m,w0m,Lm,V0m,delta_phi]=modul_na_phi(x,dt,alfa_0,alfa_1,Alfa_2,b1,b2,L_p
oprz,w0_poprz,delta_w_poprz,deltawdt_poprz,dw0dt_poprz,delta_w,w0w,delta_ph
i_poprz,f0i,q0,int_ql,int_del_ql,ql0,r1,r2,nn)

Lm=(L_poprz^(1+1/nn)+(nn+1)/nn*alfa_0^(1/nn)*dt/2*(w0w^(1+2/nn)+w0_poprz^(1
+2/nn)))^(nn/(nn+1));

cdw=sum(calka(x,delta_w_poprz,0,0,alfa_1,alfa_2))+(1/(alfa_0+1))*w0_poprz*(
1-(1-x(end))^(alfa_0+1));
cdwt=sum(calka(x,deltawdt_poprz,0,0,alfa_1,alfa_2))+(1/(alfa_0+1))*dw0dt_po
prz*(1-(1-x(end))^(alfa_0+1));
cdh=sum(calka(x,delta_w,0,0,alfa_1,alfa_2));

cql=int_ql;
FI=delta_phi_poprz+f0i*(1-x);

p1=FI(end)*(1-x(end))^alfa_0+r1*Lm/dt/(1+alfa_0)*(1-(1-x(end))^(alfa_0+1));
p2=alfa_0^(1/nn)*Lm^(-1/nn)*cdh;
p3=alfa_0^(1/nn)*Lm^(-1/nn)/(alfa_0+1)*(1-(1-x(end))^(alfa_0+1));
p4=-q0-r1*Lm/dt*cdw+r1*Lm/dt*cdh-r2*Lm*cdwt+Lm*cql;

s1=1;
s2=1+2/nn;
s3=2+2/nn;

w0m=f_z([p1 p2 p3],[s1 s2 s3],p4,w0w);

mm1=dt/(alfa_0+1)*(alfa_0^(1/nn))/(Lm^(1/nn));
mm2=dt/(alfa_0+1)*Lm*(-r2*dw0dt_poprz+ql0);
mm3=1/(alfa_0+1)*Lm*r1;

f0_poprz=f0i;
f00=(mm1*w0m^(1+2/nn)+mm2/w0m+mm3*(1-w0_poprz/w0m+abs(1-
w0_poprz/w0m)))/(dt+mm3*abs(1-w0_poprz/w0m)/f0_poprz);
wsk=abs((f0_poprz-f00)/f00);
while wsk>1e-10
f0_poprz=f00;
f00=(mm1*w0m^(1+2/nn)+mm2/w0m+mm3*(1-w0_poprz/w0m+abs(1-
w0_poprz/w0m)))/(dt+mm3*abs(1-w0_poprz/w0m)/f0_poprz);
wsk=abs((f0_poprz-f00)/f00);
end

f0m=f00;

V0m=1/(Lm^(1/nn))*alfa_0^(1/nn)*w0m^(1+2/nn);

ca1=calka(x,delta_w,0,0,alfa_1,alfa_2);
ca2=calka(x,delta_w_poprz,0,0,alfa_1,alfa_2);
ca3=int_del_ql;
```

```
for jj=1:length(x)-1
ca4(1,jj)=(1-x(jj+1))*delta_w(jj+1)-(1-x(jj))*delta_w(jj);
end
ca5=calka(x,deltawdt_poprz',0,0,alfa_1,alfa_2);

ww=delta_w+w0m*(1-x).^alfa_0;

R=Lm*r2*ca5-alfa_0^(1/nn)*Lm^(-1/nn)*(w0m)^(1+2/nn)*ca1-Lm*ca3-f0m*ca4;
F=-ca1+ca2;

bb=3;
for jj=1:length(x)-1
D(jj)=ww(jj+1)*delta_phi_poprz(jj+1)-ww(jj)*delta_phi_poprz(jj);
psi(jj)=(1-x(jj+1))^bb-(1-x(jj))^bb;
end

SD=(F+abs(F).*psi)./(dt/r1/Lm+abs(F).*psi./D)+R./(1+r1*Lm/dt*abs(F).*psi./D
);

del_1=b1;
del_2=b2;
y=1-x;
wz=y(end-2)^del_1*y(end-1)^del_2-y(end-2)^del_2*y(end-1)^del_1;
K_1=(y(end-1)^del_2*y(end)^del_1-y(end-1)^del_1*y(end)^del_2)/wz;
K_2=(y(end-2)^del_1*y(end)^del_2-y(end-2)^del_2*y(end)^del_1)/wz;

dfn=-(K_1*(SD(end)+SD(end-1))*ww(end-1)+K_2*SD(end)*ww(end-2))/(ww(end-
2)*ww(end-1)-K_1*ww(end)*ww(end-1)-K_2*ww(end)*ww(end-2));

for ii=1:length(x)-1
dfn(ii+1)=(dfn(ii)*ww(end-ii+1)-SD(end-ii+1))/ww(end-ii);
end

for ii=1:length(x)
del_fin(ii)=dfn(end-ii+1);
end

delta_phi=del_fin;

end
```

FUNCTION FOR W

```
function
[w,delta_w]=modul_na_w(x,w0,f0,V0,LL,delta_wg,wg,delta_fi,nn,alfa_0,alfa_1,
alfa_2)

dFF=(1-(1+1/w0./(1-x).^alfa_0.*delta_wg).^(nn+1))./(wg.^(nn+1));

As=f0/V0-1;

etax=(1+As*(1-x)+delta_fi/V0).^nn-1-As*nn*(1-x);

for jj=1:length(x)-1
cf1(1,jj)=1/(2-alfa_0*(nn+1))*((1-x(jj))^(2-alfa_0*(nn+1))-(1-x(jj+1))^(2-
alfa_0*(nn+1)));
end

f2=(1-x).^(-alfa_0*(nn+1)).*etax;
cf2=calka(x,f2,0,-alfa_0*(nn+1)+1,-alfa_0*(nn+1)+2,-alfa_0*(nn+1)+3);
```

```
f3=dFF.*((f0-V0)*(1-x)+delta_fi+V0).^nn;
cf3=calka(x,f3,0,-alfa_0*(nn+1),1-alfa_0*(nn+1),2-alfa_0*(nn+1));

FI=-alfa_0*w0*nn*As*cf1-alfa_0*w0*cf2-LL*cf3;

ww=0;
a0=alfa_0;
a1=alfa_1;
a2=alfa_2;

x2=x(end-1);
x4=x(end-2);
x0=x(end);

wz= (1-x2)^a2*(1-x4)^a1-(1-x2)^a1*(1-x4)^a2;
AA1=((1-x0)^a2*(1-x4)^a1-(1-x0)^a1*(1-x4)^a2)/wz;
AA2=((1-x0)^a1*(1-x2)^a2-(1-x0)^a2*(1-x2)^a1)/wz;
AA3=(1-x0)^a0+((1-x0)^a2*(1-x2)^a1*(1-x4)^a0-(1-x0)^a1*(1-x2)^a2*(1-x4)^a0-
(1-x0)^a2*(1-x2)^a0*(1-x4)^a1+(1-x0)^a1*(1-x2)^a0*(1-x4)^a2)/wz;

FF=zeros(length(x),1);
FF(end,1)=AA3*ww;
for jj=1:length(x)-1
FF(jj,1)=FI(jj);
end

MM=zeros(length(x),length(x));

for jj=1:length(x)-1
MM(jj,jj)=-1;
MM(jj,jj+1)=1;
end

MM(end,end-2)=-AA2;
MM(end,end-1)=-AA1;
MM(end,end)=1;

M=inv(MM);

zs=M*FF;

delta_w=zs';

w=w0*(1-x).^alfa_0+delta_w;

end
```

KGD VISCOSITY DOMINATED SELF SIMILAR

MAIN SCRIPT

```
clear all
clc
format long

N=100;
eps=1e-6;
n=.9;
wskaznik=1e-10;
```

```
x=sgl(N,eps);

alfa_0=2/(n+2);
alfa_1=alfa_0+1;
alfa_2=alfa_1+1;
alfa_3=alfa_2+1;
b0=1;
b1=(n+1)*alfa_0;
b2=2;

CA=alfa_0*(alfa_0-1)*cot(pi*alfa_0);

gama=1/(n+2);
eta1=-(n+2*gama*(n+2))/(n+gama*(n+2));
eta2=-(n+2)/(n+gama*(n+2));

ql=zeros(length(x),1);
ql0=0;
delta_ql=zeros(length(x),1);
int_ql=0;
int_del_ql=zeros(1,length(x)-1);

hypg=hypergeom([3/2-alfa_0,2],3/2,x.^2);
for jj=1:length(x)-1
int_hyp(jj)=(x(jj+1)*hypergeom([1/2,-alfa_0],3/2,(x(jj+1))^2)-((1-(1-
(x(jj+1))^2)^alfa_0+(x(jj+1))^2*(1-(x(jj+1))^2)^alfa_0)/(2*(1+alfa_0))))-
(x(jj)*hypergeom([1/2,-alfa_0],3/2,(x(jj))^2)-((1-(1-
(x(jj))^2)^alfa_0+(x(jj))^2*(1-(x(jj))^2)^alfa_0)/(2*(1+alfa_0)))); end

q0=1;

hg=(1-x.^2).^alfa_0;
delta_hg=(1-x.^2).^alfa_1;

[h0,f0,V0,fi,delta_fi]=blok_phi(x,hg,delta_hg,alfa_0,alfa_1,alfa_2,b0,b1,b2
,int_ql,int_del_ql,q0,ql0,eta1,eta2,CA,n,int_hyp);

[h,delta_h]=blok_h(x,hg,n,fi,V0,h0,alfa_0,hypg);

wsk=sqrt(trapz(x,(delta_hg-delta_h).^2)/trapz(x,delta_h.^2))

alk_w=.5;
alk_fi=.5;

while wsk>wskaznik
hg=alk_w*hg+(1-alk_w)*h;
delta_hg=alk_w*delta_hg+(1-alk_w)*delta_h;
clear h delta_h h0 f0 V0 delta_fi

[h0,f0,V0,fi1,delta_fi]=blok_phi(x,hg,delta_hg,alfa_0,alfa_1,alfa_2,b0,b1,b
2,int_ql,int_del_ql,q0,ql0,eta1,eta2,CA,n,int_hyp);
fi=alk_fi*fi+(1-alk_fi)*fi1;

[h,delta_h]=blok_h(x,hg,n,fi,V0,h0,alfa_0,hypg);

wsk=sqrt(trapz(x,(delta_hg-delta_h).^2)/trapz(x,delta_h.^2))

end
```

FUNCTION FOR PHI

```
function
[h0,f0,V0,phi,delta_phi]=blok_phi(x,h,delta_h,alfa_0,alfa_1,alfa_2,b0,b1,b2
,int_ql,int_del_ql,q0,ql0,eta1,eta2,ca,n,int_hyp)

int_h=calka(x,delta_h,0,0,alfa_1,alfa_2);

sumf=sum(int_h);

pp1=ca^(1/n)*(eta1*sumf+eta2*int_ql);
pt1=1+2/n;
pp2=eta1*ca^(1/n)*hypergeom([1/2,-alfa_0],3/2,1);
pt2=2+2/n;
h0=f_z([pp1 pp2],[pt1 pt2],q0,10);
f0=-((ca^(1/n)*h0^(2/n)*(eta2*ql0+eta1*h0))/(2*(1+alfa_0)));
V0=ca^(1/n)*h0^(1+2/n);

int_del_h=calka(x,delta_h,0,alfa_0,alfa_1,alfa_2);

for jj=1:length(x)-1
mh2(jj)=(1-(x(jj+1))^2)*delta_h(jj+1)-(1-(x(jj))^2)*delta_h(jj);
end

FF=eta2*V0*int_del_ql+eta1*V0*int_del_h-2*(alfa_0+1)*h0*f0*int_hyp-f0*mh2;

del_1=b1;
del_2=b2;
y=1-x.^2;
wz=y(end-2)^del_1*y(end-1)^del_2-y(end-2)^del_2*y(end-1)^del_1;
K_1=(y(end-1)^del_2*y(end)^del_1-y(end-1)^del_1*y(end)^del_2)/wz;
K_2=(y(end-2)^del_1*y(end)^del_2-y(end-2)^del_2*y(end)^del_1)/wz;

ddf=-(h(end-1)*K_1*(FF(end)+FF(end-1))+K_2*FF(end)*h(end-2))/(h(end-
1)*h(end-2)-K_1*h(end)*h(end-1)-K_2*h(end)*h(end-2));

fdf=ddf;
for ii=1:length(x)-1
fdf(ii+1)=(fdf(ii)*h(end-ii+1)-FF(end-ii+1))/h(end-ii);
end

for ii=1:length(fdf)
fdd(ii)=fdf(end-ii+1);
end

delta_phi=fdd;
phi=delta_phi+f0*(1-x.^2).^b0;

end
```

FUNCTION FOR W

```
function [h,delta_h]=blok_h(x,hg,n,fi,V0,h0,alfa_0,hypg)

dpdx=-hg.^(-(n+1)).*(abs(fi)+x*V0).^n;
gl_cz=-h0/pi*alfa_0*(2*alfa_0-1)*beta(1/2,alfa_0)*x.*hypg;
ddpdx=dpdx-gl_cz;

for ii=1:length(x)
ap=spline(x,ddpdx.*G(x(ii),x)');
```

```
app=fnint(ap);
af2(ii)=fnval(app,1)-fnval(app,0);
end

delta_h=-(4/pi)*af2;
h=h0*(1-x.^2).^alfa_0+delta_h;

end
```

KGD VISCOSITY DOMINATED TRANSIENT

MAIN SCRIPT

```
clear all
clc
format long

N=30;
eps=1e-6;
n=0.7;
wskaznik=1e-10;

x=sgl(N,eps);

alfa_0=2/(n+2);
alfa_1=alfa_0+1;
alfa_2=alfa_1+1;
alfa_3=alfa_2+1;
b0=1;
b1=(n+1)*alfa_0;
b2=2;

CA=alfa_0*(alfa_0-1)*cot(pi*alfa_0);
gama=1/(n+2);
eta1=-(n+2*gama*(n+2))/(n+gama*(n+2));
eta2=-(n+2)/(n+gama*(n+2));

hypg=hypergeom([3/2-alfa_0,2],3/2,x.^2);
for jj=1:length(x)-1
int_hyp(jj)=(x(jj+1)*hypergeom([1/2,-alfa_0],3/2,(x(jj+1))^2)-((1-(1-
(x(jj+1))^2)^alfa_0+(x(jj+1))^2*(1-(x(jj+1))^2)^alfa_0)/(2*(1+alfa_0))))-
(x(jj)*hypergeom([1/2,-alfa_0],3/2,(x(jj))^2)-((1-(1-
(x(jj))^2)^alfa_0+(x(jj))^2*(1-(x(jj))^2)^alfa_0)/(2*(1+alfa_0))));
end

MM=100;
kc=0.01;
ll=3*100/MM-2*kc;
for ii=1:MM
t(ii)=kc*(ii-1)+(1/3)*(ll-kc)/((MM-1)^2)*(ii-1)^3;
end
ts=t(1);

a=1;

for jj=1:length(x)-1
int_hyp_f(jj)=(x(jj+1)*hypergeom([1/2,-alfa_0],3/2,(x(jj+1))^2))-
(x(jj)*hypergeom([1/2,-alfa_0],3/2,(x(jj))^2));
int_2_f(jj)=(x(jj+1))^2*(1-(x(jj+1))^2)^alfa_0-(1-(x(jj+1))^2)^alfa_0-
(x(jj))^2*(1-(x(jj))^2)^alfa_0+(1-(x(jj))^2)^alfa_0;
```

```
end
hpg_w0=hypergeom([1/2,-alfa_0],3/2,x(end)^2);

w0m(1)=w0(1);
f0m(1)=f0(1);
Lm(1)=LL(1);
V0m(1)=V0(1);
fim(1,:)=fi(1,:);
delta_fim(1,:)=delta_fi(1,:);
wm(1,:)=w(1,:);
delta_wm(1,:)=delta_w(1,:);
dwdtm(1,:)=dwdt(1,:);
deltawdtm(1,:)=deltawdt(1,:);
dw0dtm(1)=dw0dt(1);

for ii=1:length(t)-1
dt=t(ii+1)-t(ii);
[w0_ost,f0_ost,L_ost,fi_ost,delta_fi_ost,w_ost,delta_w_ost,dwdt_ost,deltawd
t_ost,dw0dt_ost]=blok_czasowy(x,dt,CA,delta_wm(ii,:),deltawdtm(ii,:),w0m(ii
),dw0dtm(ii),delta_fim(ii,:),f0m(ii),alfa_0,alfa_1,alfa_2,b0,b1,b2,Lm(ii),q
0(ii+1),int_ql(ii+1),int_del_ql(ii+1,:),ql0(ii+1),wm(ii,:),dwdtm(ii,:),hpg_
w0,int_hyp_f,int_2_f,hypg,n);
w0m(ii+1)=w0_ost;
f0m(ii+1)=f0_ost;
Lm(ii+1)=L_ost;
fim(ii+1,:)=fi_ost;
delta_fim(ii+1,:)=delta_fi_ost;
wm(ii+1,:)=w_ost;
delta_wm(ii+1,:)=delta_w_ost;
dwdtm(ii+1,:)=dwdt_ost;
deltawdtm(ii+1,:)=deltawdt_ost;
dw0dtm(ii+1)=dw0dt_ost;
end
```

BLOK CZASOWY

```
function
[w0_ost,f0_ost,L_ost,phi_ost,delta_phi_ost,w_ost,delta_w_ost,dwdt_ost,delta
wdt_ost,dw0dt_ost]=blok_czasowy(x,dt,CA,delta_w_poprz,deltawdt_poprz,w0_pop
rz,dw0dt_poprz,delta_fi_poprz,f0_poprz,alfa_0,alfa_1,alfa_2,b0,b1,b2,L_popr
z,q0,int_ql,int_del_ql,ql0,w_poprz,dwdt_poprz,hpg_w0,int_hyp_f,int_2_f,hypg
,n)

r1=2;
r2=r1-1;

delta_ww=delta_w_poprz+deltawdt_poprz*dt;
ww=w_poprz+dwdt_poprz*dt;
ww0=w0_poprz+dw0dt_poprz*dt;
delta_ff=delta_fi_poprz;

[f0m,w0m1,Lm,V0m,delta_fim]=modul_na_phi(x,dt,alfa_0,alfa_1,alfa_2,b0,b1,b2
,n,CA,r1,r2,L_poprz,delta_ww,w0_poprz,ww0,w_poprz,delta_w_poprz,dwdt_poprz,
delta_ff,f0_poprz,hpg_w0,dw0dt_poprz,deltawdt_poprz,int_del_ql,int_ql,ql0,q
0,int_hyp_f,int_2_f);
w0m=w0m1;

fim=f0m*(1-x.^2).^b0+delta_fim;
```

```
[wm,delta_wm]=modul_na_w(x,ww,n,fim,V0m,w0m1,Lm,alfa_0,hypg);

wsk=sqrt(trapz(x,(delta_wm-delta_ww).^2)/trapz(x,delta_wm.^2));

alk_w=.5;
alk_fi=.5;

while wsk>1e-10
delta_ww=alk_w*delta_ww+(1-alk_w)*delta_wm;
ww=alk_w*ww+(1-alk_w)*wm;
w0m=alk_w*w0m+(1-alk_w)*w0m1;

[f0m1,w0m1,Lm1,V0m1,delta_fim1]=modul_na_phi(x,dt,alfa_0,alfa_1,alfa_2,b0,b
1,b2,n,CA,r1,r2,L_poprz,delta_ww,w0_poprz,w0m,w_poprz,delta_w_poprz,dwdt_po
prz,delta_fim,f0m,hpg_w0,dw0dt_poprz,deltawdt_poprz,int_del_ql,int_ql,ql0,q
0,int_hyp_f,int_2_f);

f0m=alk_fi*f0m+(1-alk_fi)*f0m1;
delta_fim=alk_fi*delta_fim+(1-alk_fi)*delta_fim1;
fim=f0m*(1-x.^2).^b0+delta_fim;

[wm,delta_wm]=modul_na_w(x,ww,n,fim,V0m1,w0m1,Lm1,alfa_0,hypg);

wsk=sqrt(trapz(x,(delta_wm-delta_ww).^2)/trapz(x,delta_wm.^2));

end

w0_ost=w0m1;
f0_ost=f0m;
L_ost=Lm1;
phi_ost=fim;
delta_phi_ost=delta_fim;
delta_w_ost=delta_wm;
w_ost=wm;
dwdt_ost=r1/dt*(w_ost-w_poprz)-r2*dwdt_poprz;
deltawdt_ost=r1/dt*(delta_ww-delta_w_poprz)-r2*deltawdt_poprz;
dw0dt_ost=r1/dt*(w0_ost-w0_poprz)-r2*dw0dt_poprz;

end
```

FUNCTION FOR PHI

```
function
[f0m,w0m,Lm,V0m,delta_fi]=modul_na_phi(x,dt,alfa_0,alfa_1,alfa_2,b0,b1,b2,n
,CA,r1,r2,L_poprz,delta_w,w0_poprz,w0w,w_poprz,delta_w_poprz,dwdt_poprz,del
ta_fi_poprz,f0i,hpg_w0,dw0dt_poprz,deltawdt_poprz,int_del_ql,int_ql,ql0,q0,
int_hyp,int_2_f)

Lm=(L_poprz^(1+2/n)+(n+2)/n*CA^(1/n)*dt/2*(w0w^(1+2/n)+w0_poprz^(1+2/n)))^(
n/(n+2));

cdw=sum(calka(x,delta_w,0,0,alfa_1,alfa_2));
cdwt=sum(calka(x,dwdt_poprz,0,0,alfa_0,alfa_1));
cu=sum(calka(x,w_poprz,0,0,alfa_0,alfa_1));
cql=int_ql;

FI=delta_fi_poprz+f0i*(1-x.^2).^b0;

p1=FI(end)*(1-(x(end))^2)^alfa_0+r1*Lm/dt*x(end)*hpg_w0;
```

```
p2=CA^(1/n)*Lm^(-2/n)*cdw;
p3=CA^(1/n)*Lm^(-2/n)*x(end)*hpg_w0;
p4=FI(end)*delta_w(end)-q0-r1*Lm/dt*cu+r1*Lm/dt*cdw-r2*Lm*cdwt+Lm*cql;

s1=1;
s2=1+2/n;
s3=2+2/n;

w0m=f_z([p1 p2 p3],[s1 s2 s3],p4,w0w);

f0m=1/2/(alfa_0+1)*(Lm*r1/dt-Lm*r1/dt*w0_poprz/w0m-
Lm*r2/w0m*dw0dt_poprz+CA^(1/n)*Lm^(-2/n)*w0m^(1+2/n)+Lm*ql0/w0m);

V0m=Lm^(-2/n)*CA^(1/n)*w0m^(1+2/n);

ca1=calka(x,delta_w,0,0,alfa_1,alfa_2);
ca2=calka(x,delta_w_poprz,0,0,alfa_1,alfa_2);
ca3=calka(x,deltawdt_poprz',0,0,alfa_1,alfa_2);
ca4=int_del_ql;
for jj=1:length(x)-1
ca5(1,jj)=(1-(x(jj+1))^2)*delta_w(jj+1)-(1-(x(jj))^2)*delta_w(jj);
end
ca6=int_hyp;
ca7=int_2_f;

ww=delta_w+w0m*(1-x.^2).^alfa_0;

FF=-(Lm*r1/dt+V0m)*ca1+Lm*r1/dt*ca2+Lm*r2*ca3-Lm*ca4-
2*(alfa_0+1)*f0m*w0m*ca6+f0m*w0m*ca7-f0m*ca5;

del_1=b1;
del_2=b2;
y=1-x.^2;
wz=y(end-2)^del_1*y(end-1)^del_2-y(end-2)^del_2*y(end-1)^del_1;
K_1=(y(end-1)^del_2*y(end)^del_1-y(end-1)^del_1*y(end)^del_2)/wz;
K_2=(y(end-2)^del_1*y(end)^del_2-y(end-2)^del_2*y(end)^del_1)/wz;

ddf=-(ww(end-1)*K_1*(FF(end)+FF(end-1))+K_2*FF(end)*ww(end-2))/(ww(end-
1)*ww(end-2)-K_1*ww(end)*ww(end-1)-K_2*ww(end)*ww(end-2));

fdf=ddf;
for ii=1:length(x)-1
fdf(ii+1)=(fdf(ii)*ww(end-ii+1)-FF(end-ii+1))/ww(end-ii);
end

for ii=1:length(fdf)
fdd(ii)=fdf(end-ii+1);
end
delta_fi=fdd;

end
```

FUNCTION FOR W

```
function [w,delta_w]=modul_na_w(x,wg,n,fi,V0,w0,LL,alfa_0,hypg)

dpdx=-LL*wg.^(-(n+1)).*(abs(fi)+x*V0).^n;
gl_cz=-w0/LL/pi*alfa_0*(2*alfa_0-1)*beta(1/2,alfa_0)*x.*hypg;
```

```
ddpdx=dpdx-gl_cz;

for ii=1:length(x)
ap=spline(x,ddpdx.*G(x(ii),x)');
app=fnint(ap);
af2(ii)=fnval(app,1)-fnval(app,0);
end

delta_w=-(4/pi)*LL*af2;
w=w0*(1-x.^2).^alfa_0+delta_w;

end
```

KGD TOUGHNESS DOMINATED SELF SIMILAR

MAIN SCRIPT

```
clear all
clc
format long

N=100;
eps=1e-6;
n=.6;
wskaznik=1e-10;

x=sgl(N,eps);

alfa_0=1/2;
alfa_1=(3-n)/2;
alfa_2=(5-2*n)/2;
alfa_3=alfa_1+1;
b0=1;
b1=min([2-n,(3+n)/2]);
b2=median([2-n,(3+n)/2]);

CA=alfa_1*(alfa_1-1)*cot(pi*alfa_1);
gama=n/(2+n);
eta1=-(n+2*gama*(n+2))/(n+gama*(n+2));
eta2=-(n+2)/(n+gama*(n+2));

ql=zeros(length(x),1);
ql0=0;
delta_ql=zeros(length(x),1);
int_ql=0;
int_del_ql=zeros(1,length(x)-1);
q0=1;

KI=1.1;

hypg=hypergeom([3/2-alfa_1,2],3/2,x.^2);
for jj=1:length(x)-1
int_hyp(jj)=(x(jj+1)*hypergeom([1/2,-alfa_0],3/2,(x(jj+1))^2)-((1-(1-
(x(jj+1))^2)^alfa_0+(x(jj+1))^2*(1-(x(jj+1))^2)^alfa_0)/(2*(1+alfa_0))))-
(x(jj)*hypergeom([1/2,-alfa_0],3/2,(x(jj))^2)-((1-(1-
(x(jj))^2)^alfa_0+(x(jj))^2*(1-(x(jj))^2)^alfa_0)/(2*(1+alfa_0))));
end
hypg2=hypergeom([3/2-alfa_2,2],3/2,x.^2);

h0=4/sqrt(pi)*KI;
```

```
hg=(1-x.^2).^alfa_0;
del_hg=(1-x.^2).^alfa_1;
h1p=0.8*h1b;

[h1,f0,V0,fi,del_fi]=blok_phi(x,hg,del_hg,h0,h1p,alfa_0,alfa_1,alfa_2,alfa_
3,b0,b1,b2,int_ql,int_del_ql,q0,ql0,eta1,eta2,CA,n,int_hyp);

[h,del_h]=blok_h(x,hg,n,fi,V0,h0,h1,alfa_1,alfa_2,hypg,hypg2);

wsk=sqrt(trapz(x,(del_hg-del_h).^2)/trapz(x,del_h.^2))


alk_w=.6;
alk_fi=.6;

while wsk>wskaznik
hg=alk_w*hg+(1-alk_w)*h;
del_hg=alk_w*del_hg+(1-alk_w)*del_h;
h1p=alk_w*h1p+(1-alk_w)*h1;
clear h1 f0 V0 fi1 del_fi h del_h

[h1,f0,V0,fi1,del_fi]=blok_phi(x,hg,del_hg,h0,h1p,alfa_0,alfa_1,alfa_2,alfa
_3,b0,b1,b2,int_ql,int_del_ql,q0,ql0,eta1,eta2,CA,n,int_hyp);

fi=alk_fi*fi+(1-alk_fi)*fi1;

[h,del_h]=blok_h(x,hg,n,fi,V0,h0,h1,alfa_1,alfa_2,hypg,hypg2);

wsk=sqrt(trapz(x,(del_hg-del_h).^2)/trapz(x,del_h.^2))

end
```

FUNCTION FOR PHI

```
function
[h1,f0,V0,phi,delta_phi]=blok_phi(x,h,delta_h,h0p,h1p,alfa_0,alfa_1,alfa_2,
alfa_3,b0,b1,b2,int_ql,int_del_ql,q0,ql0,eta1,eta2,ca,n,int_hyp)

delta_hs=delta_h-h1p*(1-x.^2).^alfa_1;
int_del_hs=calka(x,delta_hs,0,0,alfa_2,alfa_3);

sumf=sum(int_del_hs);

pp1=ca^(1/n)*(eta1*h0p^(2+1/n)*pi/4+eta1*h0p^(1+1/n)*sumf+eta2*h0p^(1+1/n)*
int_ql);
pt1=1/n;
pp2=eta1*ca^(1/n)*h0p^(1+1/n)*sqrt(pi)/2*gamma(alfa_1+1)/gamma(alfa_1+3/2);
pt2=1+1/n;

h1=f_z([pp1 pp2],[pt1 pt2],q0,h1p);

V0=ca^(1/n)*h0p^(1+1/n)*h1^(1/n);
f0=-eta1*V0/3-eta2*V0*ql0/3/h0p;

for jj=1:length(x)-1
mh2(jj)=(1-(x(jj+1))^2)*delta_h(jj+1)-(1-(x(jj))^2)*delta_h(jj);
end

int_del_h=calka(x,delta_h,0,0,alfa_1,alfa_2);
```

```
FF=eta2*V0*int_del_ql+eta1*V0*int_del_h-2*(alfa_0+1)*h0p*f0*int_hyp-f0*mh2;

del_1=b1;
del_2=b2;
y=1-x.^2;
wz=y(end-2)^del_1*y(end-1)^del_2-y(end-2)^del_2*y(end-1)^del_1;
K_1=(y(end-1)^del_2*y(end)^del_1-y(end-1)^del_1*y(end)^del_2)/wz;
K_2=(y(end-2)^del_1*y(end)^del_2-y(end-2)^del_2*y(end)^del_1)/wz;

ddf=-(h(end-1)*K_1*(FF(end)+FF(end-1))+K_2*FF(end)*h(end-2))/(h(end-
1)*h(end-2)-K_1*h(end)*h(end-1)-K_2*h(end)*h(end-2));

fdf=ddf;
for ii=1:length(x)-1
fdf(ii+1)=(fdf(ii)*h(end-ii+1)-FF(end-ii+1))/h(end-ii);
end

for ii=1:length(fdf)
fdd(ii)=fdf(end-ii+1);
end

delta_phi=fdd;
phi=delta_phi+f0*(1-x.^2).^b0;

end
```

FUNCTION FOR W

```
function [h,delta_h]=blok_h(x,hg,n,fi,V0,h0,h1,alfa_1,alfa_2,hypg,hypg2)

dpdx=-hg.^(-(n+1)).*(abs(fi)+x*V0).^n;

p1p=-h1/pi*alfa_1*(2*alfa_1-1)*beta(1/2,alfa_1)/gamma(3/2-
alfa_1)/gamma(2)*gamma(3/2)*gamma(2-alfa_1);
p2p=-p1p/h0*h1*(n+1);

gl_cz=-h1/pi*alfa_1*(2*alfa_1-1)*beta(1/2,alfa_1)*x.*hypg+p2p*gamma(3/2-
alfa_2)*gamma(2)/gamma(3/2)/gamma(2-alfa_2)*x.*hypg2;
ddpdx=dpdx-gl_cz;

for ii=1:length(x)
ap=spline(x,ddpdx.*G(x(ii),x)');
app=fnint(ap);
af2(ii)=fnval(app,1)-fnval(app,0);
end

h2=-pi*p2p/alfa_2/(2*alfa_2-1)/beta(1/2,alfa_2)*gamma(3/2-
alfa_2)*gamma(2)/gamma(3/2)/gamma(2-alfa_2);

ph=-(4/pi)*af2;
delta_h=h1*(1-x.^2).^alfa_1+h2*(1-x.^2).^alfa_2+ph;
h=h0*sqrt(1-x.^2)+delta_h;

end
```

KGD TOUGHNESS DOMINATED TRANSIENT

MAIN SCRIPT

```
clear all
clc
format long

N=100;
eps=1e-8;
n=0.3;
wskaznik=1e-10;

x=sgl(N,eps);

alfa_0=1/2;
alfa_1=(3-n)/2;
alfa_2=(5-2*n)/2;
alfa_3=alfa_1+1;
b0=1;
b1=min([2-n,(3+n)/2,(4-n)/2]);
b2=median([2-n,(3+n)/2,(4-n)/2]);

CA=alfa_1*(alfa_1-1)*cot(pi*alfa_1);
gama=n/(2+n);
eta1=-(n+2*gama*(n+2))/(n+gama*(n+2));
eta2=-(n+2)/(n+gama*(n+2));

for jj=1:length(x)-1
int_fp(jj)=sqrt(1-x(jj+1)^2)*(-2-3*x(jj+1)+2*x(jj+1)^2)-3*asin(x(jj+1))-
sqrt(1-x(jj)^2)*(-2-3*x(jj)+2*x(jj)^2)+3*asin(x(jj));
end
for jj=1:length(x)-1
hpg_w1_fi(jj)=x(jj+1)*hypergeom([1/2,-alfa_1],3/2,x(jj+1)^2)-
x(jj)*hypergeom([1/2,-alfa_1],3/2,x(jj)^2);
end
hpg_w0=pi/4;
hpg_w1=(sqrt(pi)*gamma(1+alfa_1))/(2*gamma(3/2+alfa_1));
hypg=hypergeom([3/2-alfa_1,2],3/2,x.^2);

MM=100;
kc=0.01;
ll=3*100/MM-2*kc;
for ii=1:MM
t(ii)=kc*(ii-1)+(1/3)*(ll-kc)/((MM-1)^2)*(ii-1)^3;
end
ts=t(1);

a=1;

w0m(1)=w0(1);
w1m(1)=w1(1);
f0m(1)=f0(1);
Lm(1)=LL(1);
V0m(1)=V0(1);
fim(1,:)=fi(1,:);
delta_fim(1,:)=delta_fi(1,:);
wm(1,:)=w(1,:);
delta_wm(1,:)=delta_w(1,:);
dwdtm(1,:)=dwdt(1,:);
deltawdtm(1,:)=deltawdt(1,:);
dw0dtm(1)=dw0dt(1);
```

```
dw1dtm(1)=dw1dt(1);
delta_wsm(1,:)=delta_ws(1,:);
deltawsdtm(1,:)=deltawsdt(1,:);
Vm(1,:)=VV(1,:);

for ii=1:length(t)-1
dt=t(ii+1)-t(ii);
r1=2;
r2=r1-1;

[w0_ost,w1_ost,f0_ost,V0_ost,L_ost,fi_ost,delta_fi_ost,w_ost,delta_w_ost,dw
dt_ost,deltawdt_ost,dw0dt_ost,dw1dt_ost]=blok_czasowy(x,dt,r1,r2,CA,delta_w
sm(ii,:),delta_wm(ii,:),deltawsdtm(ii,:),deltawdtm(ii,:),w0m(ii),w1m(ii),dw
0dtm(ii),dw1dtm(ii),fim(ii,:),f0m(ii),alfa_1,alfa_2,alfa_3,b0,b1,b2,Lm(ii),
q0(ii+1),int_ql(ii+1),int_del_ql(ii+1,:),ql0(ii+1),wm(ii,:),dwdtm(ii,:),KI(
ii+1),int_fp,hpg_w0,hpg_w1,hpg_w1_fi,hypg,n);
w0m(ii+1)=w0_ost;
w1m(ii+1)=w1_ost;
f0m(ii+1)=f0_ost;
V0m(ii+1)=V0_ost;
Lm(ii+1)=L_ost;
fim(ii+1,:)=fi_ost;
Vm(ii+1,:)=fi_ost+V0_ost*x;
delta_fim(ii+1,:)=delta_fi_ost;
wm(ii+1,:)=w_ost;
delta_wm(ii+1,:)=delta_w_ost;
dwdtm(ii+1,:)=dwdt_ost;
deltawdtm(ii+1,:)=deltawdt_ost;
dw0dtm(ii+1)=dw0dt_ost;
dw1dtm(ii+1)=dw1dt_ost;
delta_wsm(ii+1,:)=delta_w_ost-w1_ost*(1-x.^2).^alfa_1;
deltawsdtm(ii+1,:)=deltawdt_ost-dw1dt_ost*(1-x.^2).^alfa_1;
end
```

BLOK CZASOWY

```
function
[w0_ost,w1_ost,f0_ost,V0_ost,L_ost,fi_ost,delta_fi_ost,w_ost,delta_w_ost,dw
dt_ost,deltawdt_ost,dw0dt_ost,dw1dt_ost]=blok_czasowy(x,dt,r1,r2,CA,delta_w
s_poprz,delta_w_poprz,delwsdt_poprz,deltawdt_poprz,w0_poprz,w1_poprz,dw0dt_
poprz,dw1dt_poprz,fi_poprz,f0_poprz,alfa_1,alfa_2,alfa_3,b0,b1,b2,L_poprz,q
0,int_ql,int_del_ql,ql0,w_poprz,dwdt_poprz,KI,int_fp,hpg_w0,hpg_w1,hpg_w1_f
i,hypg,n)

delta_ww=delta_w_poprz+deltawdt_poprz*dt;
ww=w_poprz+dwdt_poprz*dt;

ww0=w0_poprz+dw0dt_poprz*dt;
ww1=w1_poprz+dw1dt_poprz*dt;
delta_fi_poprz=fi_poprz-f0_poprz*(1-x.^2).^b0;
delta_wsw=delta_ww-ww1*(1-x.^2).^alfa_1;

w0m1=blok_w0_red_w1_red_L(x,dt,n,CA,KI,r1,r2,delta_wsw,delwsdt_poprz,w0_pop
rz,w1_poprz,dw0dt_poprz,dw1dt_poprz,delta_ws_poprz,int_ql,ww0,ww1,q0,hpg_w0
,hpg_w1);
w0m=w0m1;

Lm=(L_poprz^(1+2/n)+(n+2)/n*CA^(1/n)*dt/2*(w0m1^(1+1/n)*ww1^(1/n)+W0_poprz^
(1+1/n)*w1_poprz^(1/n)))^(n/(n+2));
```

```
V0m=Lm^(-2/n)*CA^(1/n)*w0m^(1+1/n)*ww1^(1/n);

f0m=1/3*(Lm*r1/dt-Lm*r1/dt*w0_poprz/w0m-
Lm*r2/w0m*dw0dt_poprz+CA^(1/n)*Lm^(-2/n)*w0m^(1+1/n)*ww1^(1/n)+Lm*ql0/w0m);

delta_fim=modul_na_phi(x,dt,r1,r2,alfa_1,alfa_2,Alfa_3,b1,b2,delta_wsw,delt
a_w_poprz,deltawdt_poprz,int_del_ql,ww,w0m,ww1,Lm,V0m,f0m,int_fp,hpg_w1_fi)
;
fim=f0m*(1-x.^2).^b0+delta_fim;

w1m1=blok_w1_reg(r1,r2,dt,CA,KI,n,w0m,w0_poprz,dw0dt_poprz,ww1);
w1m=w1m1;

[wm,delta_wm,delta_wsm]=modul_na_w(x,ww,n,fim,V0m,w0m,w1m,Lm,alfa_1,hypg);

wsk=sqrt(trapz(x,(delta_wm-delta_ww).^2)/trapz(x,delta_wm.^2));

alk_w=.8;
alk_fi=.5;

while wsk>1e-10
delta_wsw=alk_w*delta_wsw+(1-alk_w)*delta_wsm;
delta_ww=alk_w*delta_ww+(1-alk_w)*delta_wm;
ww=alk_w*ww+(1-alk_w)*wm;
w0m=alk_w*w0m+(1-alk_w)*w0m1;
w1m=alk_w*w1m+(1-alk_w)*w1m1;

w0m1=blok_w0_red_w1_red_L(x,dt,n,CA,KI,r1,r2,delta_wsw,delwsdt_poprz,W0_pop
rz,w1_poprz,dw0dt_poprz,dw1dt_poprz,delta_ws_poprz,int_ql,w0m,w1m,q0,hpg_w0
,hpg_w1);

Lm=(L_poprz^(1+2/n)+(n+2)/n*CA^(1/n)*dt/2*(w0m1^(1+1/n)*w1m^(1/n)+w0_poprz^
(1+1/n)*w1_poprz^(1/n)))^(n/(n+2));

V0m1=Lm^(-2/n)*CA^(1/n)*w0m1^(1+1/n)*w1m^(1/n);

f0m1=1/3*(Lm*r1/dt-Lm*r1/dt*w0_poprz/w0m1-
Lm*r2/w0m1*dw0dt_poprz+CA^(1/n)*Lm^(-
2/n)*w0m1^(1+1/n)*w1m^(1/n)+Lm*ql0/w0m1);
delta_fim1=modul_na_phi(x,dt,r1,r2,alfa_1,alfa_2,alfa_3,b1,b2,delta_wsw,del
ta_w_poprz,deltawdt_poprz,int_del_ql,ww,w0m1,w1m,Lm,V0m1,f0m1,int_fp,hpg_w1
_fi);

f0m=alk_fi*f0m+(1-alk_fi)*f0m1;
delta_fim=alk_fi*delta_fim+(1-alk_fi)*delta_fim1;
fim=f0m*(1-x.^2).^b0+delta_fim;

w1m1=blok_w1_reg(r1,r2,dt,CA,KI,n,w0m1,w0_poprz,dw0dt_poprz,w1m);

[wm,delta_wm,delta_wsm]=modul_na_w(x,ww,n,fim,V0m1,w0m1,w1m1,
 Lm,alfa_1,hypg);

wsk=sqrt(trapz(x,(delta_wm-delta_ww).^2)/trapz(x,delta_wm.^2))

end

w0_ost=w0m1;
w1_ost=w1m1;
f0_ost=f0m;
V0_ost=V0m1;
L_ost=Lm;
```

```
fi_ost=fim;
delta_fi_ost=delta_fim;
delta_w_ost=delta_wm;
w_ost=wm;

dwdt_ost=r1/dt*(w_ost-w_poprz)-r2*dwdt_poprz;
deltawdt_ost=r1/dt*(delta_w_ost-delta_w_poprz)-r2*deltawdt_poprz;
dw0dt_ost=r1/dt*(w0_ost-w0_poprz)-r2*dw0dt_poprz;
dw1dt_ost=r1/dt*(w1_ost-w1_poprz)-r2*dw1dt_poprz;

end
```

FUNCTION FOR W0

```
function
w0m=blok_w0_red_w1_red_L(x,dt,n,CA,KI,r1,r2,delta_ws,delwsdt_poprz,w0_poprz
,w1_poprz,dw0dt_poprz,dw1dt_poprz,delta_ws_poprz,int_ql,w0w,w1m,q0,hpg_w0,h
pg_w1)

cdw=calka_spline([x 1],[delta_ws 0]);
cdwt=dw0dt_poprz*hpg_w0+dw1dt_poprz*hpg_w1+calka_spline([x
1],[delwsdt_poprz 0]);
cu=w0_poprz*hpg_w0+w1_poprz*hpg_w1+calka_spline([x 1],[delta_ws_poprz 0]);
cql=int_ql;

p2=r1/dt*pi/16/KI^2*(w1m*hpg_w1+cdw-cu)- r2*pi/16/KI^2*cdwt+pi/16/KI^2*cql;
p3=r1/dt*pi/16/KI^2*hpg_w0;
p4=CA^(1/n)*16^(2/n)*KI^(4/n)*pi^(-2/n)*(w1m^(1+1/n)*hpg_w1+w1m^(1/n)*cdw);
p5=CA^(1/n)*16^(2/n)*KI^(4/n)*pi^(-2/n)*w1m^(1/n)*hpg_w0;

s2=2;
s3=3;
s4=(n-3)/n;
s5=(2*n-3)/n;

w0m=f_z([p2 p3 p4 p5],[s2 s3 s4 s5],-q0,w0w);

end
```

FUNCTION FOR W1

```
function w1m=blok_w1_reg(r1,r2,dt,CA,KI,n,w0m,w0_poprz,dw0dt_poprz,w1w)

s=1;
m1=2/CA^(1/n)*w0m^(3/n)*(sqrt(pi)/4/KI)^(2*(2+n)/n)*(w0m-w0_poprz);
m2=-2/CA^(1/n)*w0m^(3/n)*(sqrt(pi)/4/KI)^(2*(2+n)/n)*r2*dw0dt_poprz;

w1_poprz=w1w;
w11=((r1*m1*(1+s)+m2*dt)/(dt+s*m1*r1*(w1_poprz)^(-1/n)))^n;
wsk=abs((w1_poprz-w11)/w11);
while wsk>1e-10
w1_poprz=w11;
w11=((r1*m1*(1+s)+m2*dt)/(dt+s*m1*r1*(w1_poprz)^(-1/n)))^n;
wsk=abs((w1_poprz-w11)/w11);
end

w1m=w11;
```

```
end
```

FUNCTION FOR PHI

```
function
delta_fi=modul_na_phi(x,dt,r1,r2,alfa_1,alfa_2,b1,b2,delta_w,delta_w_poprz,
deltawdt_poprz,int_del_ql,ww,w0m,Lm,V0m,f0m,int_fp)

cw=calka(x,delta_w,0,0,alfa_1,alfa_2);
cu=calka(x,delta_w_poprz,0,0,alfa_1,alfa_2);
cdwt=calka(x,deltawdt_poprz',0,0,alfa_1,alfa_2);
cql=int_del_ql;
for jj=1:length(x)-1
ca5(1,jj)=(1-(x(jj+1))^2)*delta_w(jj+1)-(1-(x(jj))^2)*delta_w(jj);
end
FF=-(Lm*r1/dt+V0m)*cw+Lm*r1/dt*cu+Lm*r2*cdwt-Lm*cql+1/2*f0m*w0m*int_fp-
f0m*ca5;

del_1=b1;
del_2=b2;
y=1-x.^2;
wz=y(end-2)^del_1*y(end-1)^del_2-y(end-2)^del_2*y(end-1)^del_1;
K_1=(y(end-1)^del_2*y(end)^del_1-y(end-1)^del_1*y(end)^del_2)/wz;
K_2=(y(end-2)^del_1*y(end)^del_2-y(end-2)^del_2*y(end)^del_1)/wz;
ddf=-(ww(end-1)*K_1*(FF(end)+FF(end-1))+K_2*FF(end)*ww(end-2))/(ww(end-
1)*ww(end-2)-K_1*ww(end)*ww(end-1)-K_2*ww(end)*ww(end-2));

fdf=ddf;
for ii=1:length(x)-1
fdf(ii+1)=(fdf(ii)*ww(end-ii+1)-FF(end-ii+1))/ww(end-ii);
end

for ii=1:length(fdf)
fdd(ii)=fdf(end-ii+1);
end

delta_fi=fdd;

end
```

FUNCTION FOR W

```
function [w,delta_w,delta_ws]=modul_na_w(x,wg,n,fi,V0,w0,w1,LL,alfa_1,hypg)

dpdx=-LL*wg.^(-(n+1)).*(abs(fi)+x*V0).^n;
gl_cz=-w1/LL/pi*alfa_1*(2*alfa_1-1)*beta(1/2,alfa_1)*x.*hypg;
ddpdx=dpdx-gl_cz;

for ii=1:length(x)
ap=spline(x,ddpdx.*G(x(ii),x)');
app=fnint(ap);
af2(ii)=fnval(app,1)-fnval(app,0);
end

delta_ws=-(4/pi)*LL*af2;
delta_w=w1*(1-x.^2).^alfa_1+delta_ws;
w=w0*sqrt(1-x.^2)+delta_w;
end
```