Bus bridging for rail disruptions

Yang, Ming ; Ma, Hongguang; Li, Xiang; Shang, Changjing; Shen, Qiang

Published in:
IEEE Transactions on Fuzzy Systems

DOI:
10.1109/TFUZZ.2022.3224789

Publication date:
2023

Citation for published version (APA):
Yang, M., Ma, H., Li, X., Shang, C., & Shen, Q. (2023). Bus bridging for rail disruptions: A distributionally robust fuzzy optimization approach. *IEEE Transactions on Fuzzy Systems*, *31*(2), 500-510. https://doi.org/10.1109/TFUZZ.2022.3224789

Bus bridging for rail disruptions: A distributionally robust fuzzy optimization approach

Ming Yang, Hongguang Ma, Xiang Li, Changjing Shang, and Qiang Shen

***Abstract*—Dealing with uncertain rail disruptions effectively raises a significant challenge for computational intelligence research. This paper studies the bus bridging problem under demand uncertainty, where the passenger demand is represented as parametric interval-valued fuzzy variables and their associated uncertainty distribution sets. A distributionally robust fuzzy optimization model is proposed to minimize the maximum travel time and to search for the optimal scheme for vehicle allocation, route selection, and frequency determination. To solve the proposed robust model, we discuss the computational issues concerning credibilistic constraints, turning the robust counterpart model into computationally tractable equivalent formulations. The proposed approach is verified, and the resulting method is validated with a report on uncertain parameters in a real-world disrupted event of Shanghai Rail Line 1. Experiment results show that the distributionally robust fuzzy optimization approach can provide a better uncertainty-immunized solution.**

***Index Terms*—Rail disruptions, Bus bridging, Distributionally robust, Credibilistic optimization, Type-2 fuzzy set.**

I. Introduction

THE railway is a sustainable transport system, which effectively reduces congestion and carbon emission. In 2020, transport volume of rail transit in major Chinese cities, e.g., Beijing, Shanghai, and Guangzhou, occupied more than 50% of the total public transportation volume[1]. However, the rail transit system often suffers from unexpected operational problems and accidents, leading to operation disruptions. Such a disruption may cause partial or entire rail system closure, which may block traffic and become unsustainable. For example, in 2018, a signal failure on Shanghai Rail Line 2 led to over five-hour delays and about 56,000 passengers stranded in affected stations during the morning rush[2]. Meanwhile, overcrowding on the rail platforms can create potentially highly risky or even dangerous situations for passengers. Therefore, it is necessary to timely evacuate stranded passengers from disruptive rail transit stations efficiently to ensure the rail system's sustainability.

To reduce the overcrowding caused by rail disruptions, rail system operators usually utilize a bus bridging strategy to evacuate stranded passengers. This strategy focuses on dispatching buses to pre-designed routes to restore and maintain the connectivity along the affected rail stations [1], [2]. The core decisions regarding bus bridging are dispatching buses, designing temporary bridging routes, and determining bridging frequency. Given a disruption situation, one general approach is to dispatch buses to the turnover stations and run them in parallel to the entire group of affected stations between two turnover stations. However, for any situations involving a non-uniform passenger demand, the conventional full-length mode may result in inefficient scheduling and operation of the buses. A short-turn mode is desirable to bridge any two stations with intermediate stations. In addition, an express mode may stop at major stations but skip some minor stations to improve high passenger demand between major stations. Thus, to increase the flexibility of bus service and reduce the passengers' travel time, we consider a mix of express and local service modes for both full-length and short-turn routes concerning bus bridging.

In real-world situations, the bus bridging problem is typically characterized by a high degree of commuter demand uncertainty, which exerts an important influence on policies or strategies adopted for the bus dispatching, route selection, and determination of frequency. The majority of existing bus bridging studies require the complete distributional information of demand data. This means the complete trip distributions should be fixed in advance and are perfectly clear. However, demand for mass commuting load in a short time does not equate to those raised during daily traffic events. The exact possible distribution of demand is usually unavailable due to the lack of historical data. Instead, the uncertain information is ambiguous and has to be estimated based on past experiences or by the subjective judgment of experts. It's reasonable to assume that the exact information is embodied in an interval area, so we consider the interval-valued fuzzy variable to characterize the uncertain demand. To characterize different fluctuation modes of uncertain demand, the lambda selection variable is introduced as the representative of a parametric interval-valued fuzzy variable.

In addition, when only partial distribution information of uncertain parameter is available, the decision maker may wish to gain a feasible solution for all the possible distributions. Based on the idea of credibilistic optimization [3]–[5], this paper develops a distributionally robust fuzzy optimization approach for bus bridging, in which the possible perturbation

Manuscript received April 19, XXXX; revised August 26, XXXX. This work was supported by grants from the National Natural Science Foundation of China (Nos. 71722007 & 71931001), the Key Program of NSFC-FRQSC Joint Project (NSFC No. 72061127002 and FRQSC No. 295837), the Funds for First-class Discipline Construction (XK1802-5), the Fundamental Research Funds for the Central Universities (buctrc201926), and the Strategic Partnership Acceleration Award (80761-Au210), funded under the Ser Cymru II programme, UK.

Ming Yang, Hongguang Ma, and Xiang Li (Corresponding Author, E-mail: lixiang@mail.buct.edu.cn) are with the School of Economics and Management, Beijing University of Chemical Technology, Beijing 100029, China.

Changjing Shang and Qiang Shen are with the Department of Computer Science, Aberystwyth University, Aberystwyth SY23 3DB, U.K.

[1]https://www.163.com/dy/article/G8KB6DKG0515APP6.html.

[2]https://www.shine.cn/news/metro/1804263729/.

of uncertain distribution is described by an uncertainty distribution set [6]. The objective is to minimize the maximum bridging time. In achieving this objective under uncertain situations, credibilistic constraints are reformulated into their equivalent robust counterpart representations. This results in a novel approach that successfully resolves the challenging problem of bus bridging for overcrowding caused by rail disruptions.

The main contributions of this paper are summarized as follows:

- A bus bridging service plan is developed that addresses a mix of express and local service modes for both full-length and short-turn routes.
- A new distributionally robust fuzzy optimization approach is proposed for the bus bridging problem, in which the uncertain demand is described as a parametric interval-valued fuzzy variable. This approach could satisfy the requirement that a solution has to be feasible for all realizations of the distributions concerned.
- A real-world case is studied to demonstrate the effectiveness of the proposed approach, addressing Shanghai Rail Line 1, with the objective being to bridge the passengers from disrupted stations to their destination stations in a minimum clear time. The computational results illustrate that the proposed distributionally robust fuzzy optimization model outperforms the nominal model under uncertain demands.

The rest of this paper is organized as follows. In Section 2, we present a literature review on the general bus bridging problem under uncertain environments. In Section 3, we describe the specific problem and present the new distributionally robust fuzzy bus bridging approach. In Section 4, we analyze the distributionally robust optimization approach and turn credibilistic constraints into a computationally tractable formulation. Section 5 presents the case study to illustrate the effectiveness of the proposed distributionally robust optimization approach. The conclusions and future research are then presented in the final Section.

II. Literature Review

A number of studies have been conducted on bus bridging under uncertainty in the literature relevant to our work. In particular, a sequence of critical problems is generally focused on developing networks, dispatching buses, designing bus routes, and determining frequency. Kang *et al.* [7] developed a last train timetable optimization and bus bridging problem for an urban railway transit network that consists of a set of rail lines. The objective of the bus bridging service was to minimize the waiting time of passengers. A decomposition method was used to resolve large-scale problems. Jin *et al.* [8] presented a path-based multicommodity network flow model for improving the bus bridging service, where the network consists of non interchange and interchange rail stations. The objective was to minimize the total travel time of all passengers.

The problem on bus bridging routes is usually designed for local mode or express mode, where the local mode stops at all specified stations and express mode only provides for several major stations [9]. For example, Van *et al.* [10] studied local route bus bridging planning for link closures concerning the selection of shuttle bus lines and frequencies under their budget constraints. The objective is to minimize operating costs and passenger inconvenience. Wang *et al.* [11] considered a feeder-bus dispatch planning model with the local bridging route. The model's goal is to minimize feeder buses' total traveling time when targeting the problem of stranded passenger evacuation. Hu *et al.* [12] presented an express route to schedule a bus bridging strategy to evacuate passengers from disrupted stations to turnover stations on one rail line. Considering minimizing the maximum evacuation time for all disrupted stations, the model determined the dispatched station and the number of roundtrips for each bus.

To solve the rail station disruption problem, Yin *et al.* [13] explored the use of an express route service where bridging buses are operated between the disrupted station and the rest of the rail stations. The objective is to minimize the total travel time of affected passengers and the operating cost of the bridging buses. Moreover, several studies also considered local and express bus bridging services as candidate routes for non-uniform passenger demand. An example of such work is that reported by [14], proposing a two-stage model to optimize the bridging strategy bus bridging plan, where the first stage was to minimize bus bridging time and the second stage was to minimize the total passenger delay. The resulting model assigned buses to both local and express bus bridging routes. In their work, the local routes only connected the disrupted stations, and the express routes were operated between the disrupted station and the turnover station. However, because there are still major stations with large passenger flow in the interrupted stations, it is difficult to meet the needs of rapidly evacuating a large number of passengers by interrupting the local services between stations. Therefore, this paper studies the hybrid bus bridge problem in which local and express route services can be bridged between all stations (including disrupted and turnover stations).

Other researchers and practitioners in bus bridging service problems are concerned with uncertainty in the problem data. Problems modeled with stochastic programming are often solved by exploiting the exact distribution assumed about their underlying random variables. Luo and Xu [15] proposed a stochastic programming model for the design of bus bridging services that involve uncertain commuter demands and spare capacities of existing rail and bus lines. The objective of their model is to minimize expected unsatisfied commuter demand under rail transit disruptions. However, in reality, information on exact distribution is often difficult to obtain, and only partial or even no distribution information is available by exploiting historical data. In this case, it is better to deal with the uncertain demand as fuzziness based on experiences or subjective judgments of experts. Type-2 fuzzy sets, as an extension of type-1 fuzzy sets, are good at modeling the uncertainty embedded in secondary possibility distributions [4], [16]–[18]. The type-2 fuzzy theory has been successfully applied to many fields, such as facility location selection [19], comparative linguistic expressions [20] and transportation problem [21]. In addition, to make the decisions more flexible and practical, Liu and Liu [4] proposed the lambda selections of parametric interval-valued fuzzy to describe the uncertain parameter.

Due to the ambiguity of expert opinions and the impact of potential external environments, the decision maker may wish to use a reliable strategy that immunizes against the perturbation of distribution information. Robust optimization offers a significant potential to find solutions that still perform well [22], [23]. For example, Liang *et al.* [24] focused on developing a plan for robust bus bridging service in response to rail transit system disruptions by considering bus travel time uncertainty. The results showed that the robust solution could remain feasible even when the bus travel time deviates to the worst case. Kulshrestha *et al.* [25] proposed a transit-based evacuation model under the uncertainty of demand, leading to a robust optimization approach for determining the optimal decisions for producing a reliable plan. Other relevant robust optimization methods can be formed in [26]–[28].

Motivated by the above observation, we herein develop a distributionally robust optimization approach, which is novel in dealing with uncertain bus bridging problems. In our model, the demands of passengers in each disrupted station are represented with uncertain parameters, which can be specified using expert opinions. To the best of our knowledge, this is the first time such an approach is proposed to handle bus bridging.

III. Bus bridging optimization model

In this section, we propose an optimization model to identify the best bus bridging service plan, considering demand uncertainty. We first introduce the problem of bus bridging with the underlying assumptions in the context of urban transit rail networks. Then, we illustrate each part of the model formulations, including the decision variables, objective function and constraints. Significantly, uncertain information on passenger demand in terms of fuzziness is formulated in the model.

A. Problem statement

We consider a general scenario that an emergency event on a single-direction rail line has resulted in the closure of several stations. The rail line is disconnected between two turnover stations, while short routing operations are possible beyond the two. Buses are used to connect disrupted stations and transport passengers to destination stations or turnover stations. As illustrated in Figure 1, a rail link between two turnover stations TS1 and TS2 is disrupted. Consequently, many passengers are affected and stay in these stations. Among them, some passengers choose alternative ways to reach their destinations, while most passengers have to stand at the station to wait for help because of the limited capacity of other transportation modes. In this case, a bus bridging service is used to travel disrupted passengers from their strand place to destination stations.

A common strategy is to completely replace disrupted rail service with bus bridging circulating between the two turnover stations. However, due to the imbalance in the number of passengers in each station, the bridging process of all-station is not effective. Therefore, using partial stations as a short-turn operation is an effective strategy. For example, as Figure 1 shows, there are two types of disrupted stations, including major station (i.e., DS2) and minor station (i.e., DS1 and DS3), where a major station refers to one with high pedestrian traffic, such as rail stations in commercial areas, transfer stations, etc. If many passengers travel between stations TS1 and DS1, building a short-turn route between these two stations is a convenient and effective strategy. Meanwhile, the local and express modes are also considered, where the local mode stops at all stations (i.e., route BS1-BS2-BS3) and the express mode only transits between the two major stations (i.e., route BS1-BS3) [9]. For easy operation, the bridged bus can only serve the local or express routes.

The sequence of proposed bus bridging service includes the dispatching and bridging processes. During the dispatching process, the buses originally depart from the depot and run to the demand stations. After picking up the passengers, the buses leave the rail station to the nearest bus station, travel through a series of bus stations, and repeatedly operate between the bridging stations several times. To facilitate the model formulation, we make the following assumptions:

1. Passengers can get off the bus at each station but can only get on from the starting station. This is because the rail station is generally far from the nearest bus station, and it is inconvenient for buses to transfer between the rail station and the bus station in an emergency period.

2. The bus dispatching time and bridging time between bus stations is deterministic, which can be estimated using historical data.

3. Each station has sufficient space for the bridged buses to stop.

To better understand the proposed strategy of the bus bridging service, let us consider a simple illustrative example, as shown in Figure 1. There are four service stations (TS1, DS1, DS2 and DS3), four destination stations (DS1, DS2, DS3 and TS1), and two vehicles with a capacity of 5 each. The dispatching time and inter-station running time are 2 time units, and the service time at each station is one time unit. For easy illustration, only station TS1 generates the demand for different destinations. Note that although TS1 is not a disrupted station, there are also a large number of passengers who are unable to commute due to the disrupted link. The sampled demands associated with station TS1 and the optimal operation strategy are listed in Table I.

As shown in Table I, bus 1 is dispatched from the depot to the station TS1 first (3 time units) and then transports passengers over the local route TS1-DS1-DS2-DS3 in sequence (9 time units). The roundtrip of this route is 2, and the corresponding travel time is 32 time units. Then, the total travel time of bus 1 is 44 time units. Bus 2 is assigned to the express route (3 time units) and is bridged from station TS1 to station DS2 (5 time units). Bus 2 makes 3 roundtrips on this express route, and the corresponding travel time is 30 time units. Then, the total travel time of bus 2 is 38 time units. Note that a roundtrip means a trip to a destination station and back usually over the same route. The total travel time is equal to the sum of dispatching time, bridging time and roundtrip

TABLE I
EXAMPLE OF A BUS BRIDGING PLAN

Route	Demands	Travel mode	Roundtrips	Bus index	Time
TS1-DS1	4	Local	2	1	44
TS1-DS2	26	Express	3	2	38
TS1-DS3	3	Local	2	1	44

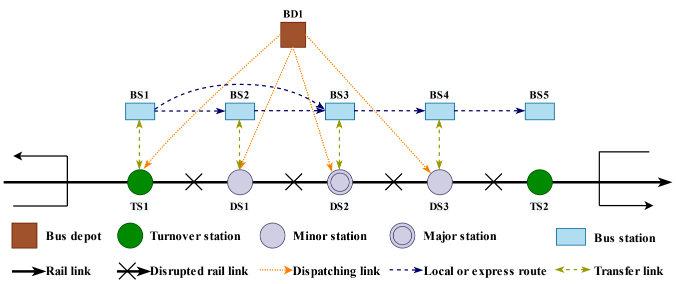

Fig. 1. Illustration of bus bridging routes on a rail line.

TABLE II
NOTATIONS

Sets and parameters	
B	Set of vehicles
D	Set of depots
S	Set of stations, including disrupted and turnover stations
A	Set of OD pairs
d_{ij}	Passenger demand from station i to station j
t_{ij}	Bus running time from station i to station j
f_i	Bus dispatching time from the depot to the disrupted station
s_i	Boarding or alighting time at station i
C	Capacity of vehicle
Intermediate variables	
T^b	Total travel time for bus b
T_{max}	Bus clear evacuation time
v_l^b	The bridging time on a local route for bus b
u_l^b	The roundtrip time on a local route for bus b

time.

B. Model formulation for the bridging

For the convenience of the readers, the notations used throughout this paper are summarized in Table II.

■ *Decision variables*

Presently, the decision variables are related to bus dispatching and bus bridging. Therefore, they can be divided into two groups, which are as follows.

1) Dispatching variables:

- x^b_{ij}: A binary variable indicating whether bus b is assigned to an express route, dispatched from station i to station j. If so, $x^b_{ij} = 1$, otherwise $x^b_{ij} = 0$.
- y^b_i: A binary variable indicating whether bus b is assigned to a local route and dispatched to station i. If so, $y^b_i = 1$, otherwise $y^b_i = 0$.
- z^b_{ij}: A binary variable indicating whether bus b is assigned to a local route connecting station i and station j. If so, $z^b_{ij} = 1$, otherwise $z^b_{ij} = 0$.

2) Bridging variables:

- m^b_{ij}: An integer variable indicating the number of roundtrips on an express route for bus b, bridged from station i to station j.
- n^b_{ij}: An integer variable indicating the number of roundtrips on a local route for bus b connecting station i and station j.
- pe^b_{ij}: Number of passengers from station i to station j boarding on an express route for bus b.
- pl^b_{ij}: Number of passengers from station i to station j boarding on a local route for bus b.

■ *Objective function*

The objective function formulated in this study aims to minimize the clear evacuation time, i.e., minimizing the maximum evacuation time for all buses. The evacuation time consists of three components: dispatching time, bridging time and deadhead time. Specifically, the dispatching time means the travel time of a vehicle from the depot to the disrupted station. The bridging time refers to the travel time of a vehicle from the origin disrupted station to the destination station to evacuate passengers. The deadhead time implies the travel time when a vehicle is not carrying passengers. Generally, this occurs when the vehicle returns from the destination station to the origin station. Accordingly, the travel time equations on express and local routes appear in the following form respectively:

$$T^b_e = \sum_{(i,j)\in A} (f_i + t_{ij} + s_i + s_j) \times x^b_{ij} + \sum_{(i,j)\in A} (t_{ij} + t_{ji} + s_i + s_j) \times m^b_{ij}, \ \forall b \in B \quad (1)$$

$$T^b_l = \sum_{i\in S} f_i \times y^b_i + \max_{(i,j)\in A} (t_{ij} + \sum_{k=i}^{j} s_k) \times z^b_{ij} + \max_{(i,j)\in A} (t_{ij} + t_{ji} + \sum_{k=i}^{j} s_k) \times n^b_{ij}, \ \forall b \in B \quad (2)$$

where f_i represents the dispatching time of a bus from depot to the disrupted station, t_{ij} represents the running time from station i to station j, s_i represents the boarding or alighting time at station i. Note that under the express mode, a bus does not stop at intermediate stations, while in the local mode, buses need to stop during the bridging process but do not need to stop during a deadhead process. Therefore, $t_{ij} + s_i + s_j$ and $t_{ij} + t_{ji} + s_i + s_j$ mean the bridging time and roundtrip time on an express route, respectively. $t_{ij} + \sum_{k\in\{i,\cdots,j\}} s_k$ and $t_{ij} + t_{ji} + \sum_{k\in\{i,\cdots,j\}} s_k$ mean the bridging time and roundtrip time on a local route, respectively.

Then, the travel time for bus b can be expressed by

$$T^b = T^b_e + T^b_l, \ \forall b \in B. \quad (3)$$

Based on the above, the goal of the optimization model is an effort to minimize the duration of the evacuation that is specified by the highest travel time of any bus can be written as follows:

$$\begin{aligned} \min \quad & T_{max} \\ \text{s.t.} \quad & T_{max} \geq T^b, \ \forall b \in B, \end{aligned} \quad (4)$$

where Constraint (4) requires T_{max} to be greater than or equal to the maximum travel time incurred by any bus, which is then minimized by the objective function.

■ *Constraints*

Constraints associated with route dispatch and passenger assignment are described here. In particular, Constraint (5) guarantees that a bus can be dispatched either to an express route or to a local route. Constraints (6) and (7) ensure that bus b can make roundtrips exclusively on the express or the local route only, respectively. Specifically, only when the bus is assigned to a certain route can it makes roundtrip decisions.

$$\sum_{i\in S} y_i^b + \sum_{(i,j)\in A} x_{ij}^b \leq 1,\ \forall b \in B \tag{5}$$

$$m_{ij}^b \leq M \times x_{ij}^b,\ \forall (i,j) \in A, b \in B \tag{6}$$

$$n_{ij}^b \leq M \times z_{ij}^b,\ \forall (i,j) \in A, b \in B, \tag{7}$$

where M is a certain large number. Constraints (8) and (9) ensure that bus b bus b runs over the same roundtrips from station i to station j if it is dispatched to a local route. Constraint (10) means that bus b can serve the local route (i,j) only if it is dispatched to station i:

$$n_{ij'}^b \geq n_{i,i+1}^b - M*(1-z_{ij}^b),\quad \forall (i,j)\in A, j'=i+1,i+2,\cdots,j, b\in B \tag{8}$$

$$n_{ij'}^b \leq n_{i,i+1}^b + M*(1-z_{ij}^b),\quad \forall (i,j)\in A, j'=i+1,i+2,\cdots,j, b\in B \tag{9}$$

$$z_{ij}^b \leq y_i^b,\ \forall i,j \in S, b\in B. \tag{10}$$

Finally, Constraint (11) indicates that the roundtrips of bus b should satisfy the number of passengers connecting station i and station j on an express route. Constraint (12) ensures that the roundtrips of bus b should satisfy the number of passengers from station i to station j on a local route. Constraint (13) ensures that all passengers between disrupted stations are transported.

$$C\times(m_{ij}^b + x_{ij}^b) \geq pe_{ij}^b,\ \forall (i,j)\in A, b\in B \tag{11}$$

$$C\times(n_{i,i'+1}^b + z_{i,i'+1}^b) \geq \sum_{k=i'+1}^{j} pl_{ik}^b,\quad \forall (i,j)\in A, i'=i,\cdots,j-1, b\in B \tag{12}$$

$$\sum_{b\in B}(pe_{ij}^b + pl_{ij}^b) = d_{ij},\ \forall (i,j)\in A. \tag{13}$$

C. Distributionally robust fuzzy bus bridging model

Passenger demand is often uncertain in dealing with a bus bridging problem, which may come from various features such as traffic and weather conditions. Because of the noise in historical data or the ambiguity of expert opinions, it is difficult to obtain precise descriptions of the passenger demand for every pair of stations. In this section, we develop a parametric credibilistic optimization model to seek the best bus bridging while considering uncertainty in passenger demands.

For this purpose, the uncertain demand $\tilde{d}_{ij}$ is assumed to be an interval-valued fuzzy random variable [29] with a possibility distribution. In addition, decision makers typically require that the credibility of this constraint meet a certain confidence level. Thus, for any $(i,j)\in A$, the credibility constraint is set such that

$$\mathrm{Cr}\left\{\sum_{b\in B}(pe_{ij}^b + pl_{ij}^b) \geq \tilde{d}_{ij}\right\} \geq \beta_{ij},\ \forall (i,j)\in A, \tag{14}$$

where the credibility constraint implies that the passenger demand is less than the threshold regarding the number of passengers allowed on board at each station, at least with a probabilistic confidence level β.

Based on the definition in [4], let ζ_{ij} be a parametric interval-valued fuzzy variable of $\tilde{d}_{ij}$ with the secondary possibility distribution $\mu_\zeta(d_{ij}) = [\mu_{\zeta^L}(d_{ij};\theta_l), \mu_{\zeta^R}(d_{ij};\theta_r)]$, where $\mu_{\zeta^L}(d_{ij};\theta_l)$ is the lower parametric possibility distribution and $\mu_{\zeta^R}(d_{ij};\theta_r)$ the upper one. In order to describe the perturbation of the possibility distribution $\mu_\zeta(d_{ij})$, fuzzy variables $\zeta_{ij,\lambda}$ are introduced to define the so-called λ selection variables of parametric interval-valued fuzzy variables for any $\lambda_{ij}\in[0,1]$. Then, the parametric possibility distribution can be characterized as follows

$$\mu_{\zeta^\lambda}(d_{ij};\theta) = (1-\lambda_{ij})\mu_{\zeta^L}(d_{ij};\theta_l) + \lambda_{ij}\mu_{\zeta^R}(d_{ij};\theta_r),\quad \text{where } \theta=(\theta_l,\theta_r), \forall (i,j)\in A. \tag{15}$$

Note that in practical applications, the values of θ_l and θ_r can be determined by the decision maker based on the experts' experiences or subjective judgments.

According to this formulation, the change of the λ parameter determines the location of the parametric possibility distribution. That is, the credibility constraint above is a collection of constraints that are of a common structure (i.e., with a fixed parameter λ), with the parameter varying in a given uncertainty distribution set $\mathcal{U}_\zeta$, which is defined by

$$\mathcal{U}_\zeta = \{\mu_{\zeta^\lambda}(d_{ij};\theta) | \mu_{\zeta^\lambda}(d_{ij};\theta) \text{ is determined by Equation (15), where } \lambda_{ij}\in[0,1], \forall (i,j)\in A\}. \tag{16}$$

From this uncertainty distribution set, the distribution robust emergency evacuation model can be expressed as follows:

$$\begin{aligned} \min \quad & T_{max} \\ \text{subject to} \quad & \mathrm{Cr}\left\{\sum_{b\in B}(pe_{ij}^b + pl_{ij}^b) \geq \zeta_{ij}\right\} \geq \beta_{ij}, \\ & \forall (i,j)\in A, \mu_{\zeta^\lambda}(d_{ij};\theta)\in\mathcal{U}_\zeta \\ & \text{Constraints} \quad (1)-(12). \end{aligned} \tag{17}$$

The above objective function is to minimize the duration of the evacuation. As such, this model focuses on a semi-infinite programming problem, and it is severely intractable [30]. In the next section, we will turn the credibility constraints into their equivalent deterministic ones to facilitate the search for a feasible optimal solution to the problem.

IV. Analysis of Bus Bridging Optimization Model

This section aims to turn the credibility bus bridging optimization model into a computationally tractable formulation. Decision makers in the problem domain tend to choose d-ifferent parametric interval-valued fuzzy variables according to different given situations. It is therefore assumed that the uncertain demand ζ_{ij} is rendered as a parametric interval-valued trapezoidal fuzzy variable. From this, the analytical expression of the bus bridging model is discussed below.

A. Analysis of credibilistic constraint

When the passenger demand follows a trapezoidal distribution, we can represent the analytical expression of credibility constraint (17) in the following theorem.

Theorem 1: Let evacuation demand $\zeta_{ij} = [r_1^{ij}, r_2^{ij}, r_3^{ij}, r_4^{ij}; \theta_l^{ij}, \theta_r^{ij}]$ be a parametric interval-valued trapezoidal fuzzy variable. If ζ_{ij} are mutually independent, then the credibility constraint concerned can be transformed into the following:

(i) If $\beta_{ij} \in (0, \frac{\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij}+1}{4}]$, then the credibility constraint is equivalent to

$$\frac{2\beta_{ij}r_2^{ij} + [\lambda_{ij}\theta_r^{ij} - (1-\lambda_{ij})\theta_l^{ij} + 1 - 2\beta_{ij}]r_1^{ij}}{1 + \lambda_{ij}\theta_r^{ij} - (1-\lambda_{ij})\theta_l^{ij}} \leq \omega_{ij}.$$

(ii) If $\beta_{ij} \in (\frac{\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij}+1}{4}, \frac{1}{2}]$, then the credibility constraint is equivalent to

$$\frac{[2\beta_{ij} - \lambda_{ij}\theta_r^{ij} + (1-\lambda_{ij})\theta_l^{ij}]r_2^{ij} + (1-2\beta_{ij})r_1^{ij}}{1 - \lambda_{ij}\theta_r^{ij} + (1-\lambda_{ij})\theta_l^{ij}} \leq \omega_{ij}.$$

(iii) If $\beta_{ij} \in (\frac{1}{2}, \frac{3-\lambda_{ij}\theta_r^{ij}+(1-\lambda_{ij})\theta_l^{ij}}{4}]$, then the credibility constraint is equivalent to

$$\frac{(1-2\beta_{ij})r_4^{ij} + [\lambda_{ij}\theta_r^{ij} - (1-\lambda_{ij})\theta_l^{ij} - 2 + 2\beta_{ij}]r_3^{ij}}{\lambda_{ij}\theta_r^{ij} - (1-\lambda_{ij})\theta_l^{ij} - 1} \leq \omega_{ij}.$$

(iv) If $\beta_{ij} \in (\frac{3-\lambda_{ij}\theta_r^{ij}+(1-\lambda_{ij})\theta_l^{ij}}{4}, 1]$, then the credibility constraint is equivalent to

$$\frac{[\lambda_{ij}\theta_r^{ij} - (1-\lambda_{ij})\theta_l^{ij} - 1 + 2\beta_{ij}]r_4^{ij} + (2-2\beta_{ij})r_3^{ij}}{1 + \lambda_{ij}\theta_r^{ij} - (1-\lambda_{ij})\theta_l^{ij}} \leq \omega_{ij}.$$

where $\omega_{ij} = \sum_{b\in B}(pe_{ij}^b + pl_{ij}^b)$.

For presentational simplicity, we use $\pi_\beta(d;\theta,\lambda)$ to express the above piecewise functions collectively and denote D_β^n as the definition domain of each segment function. Then the credibility constraint (17) can be expressed as

$$\pi_\beta(d;\theta,\lambda) \leq \sum_{b\in B}(pe_{ij}^b + pl_{ij}^b),\ \forall(i,j)\in A, \beta_{ij}\in D_\beta^n. \quad (18)$$

The proof of this theorem is given in Appendix A.

B. Analysis of robust counterpart

After turning the credibility constraints into their equivalent formulations, the perturbation of the parametric interval value possibility distribution can be resolved. With a predefined uncertainty distribution set $\mathcal{U}_\zeta$, the aim is to find the solutions that remain feasible for any possible distribution, that is

$$\max_{\mu_{\zeta\lambda}(d_{ij};\theta)\in\mathcal{U}_\zeta} \{\pi_\beta(d;\theta,\lambda)\} \leq \sum_{b\in B}(pe_{ij}^b + pl_{ij}^b),$$
$$\forall(i,j)\in A,\ \beta_{ij}\in D_\beta^n, \quad (19)$$

To resolve the above maximization problem, we use the theorem of duality [31] to transform the left hand maximization into its conic dual. As a consequence, inequality (19) becomes equivalent to the following piecewise function

$$\pi_\beta^*(d;\theta) = \begin{cases} \frac{2\beta_{ij}r_2^{ij} + [1-\theta_l^{ij}-2\beta_{ij}]r_1^{ij}}{1-\theta_l^{ij}} \leq \omega_{ij}, & \beta_{ij}\in(0, \frac{1-\theta_l^{ij}}{4}] \\ \frac{[2\beta_{ij}+\theta_l^{ij}]r_2^{ij} + (1-2\beta_i^t)r_1^{ij}}{1+\theta_l^{ij}} \leq \omega_{ij}, & \beta_{ij}\in(\frac{1-\theta_l^{ij}}{4}, \frac{1}{2}] \\ \frac{(1-2\beta_{ij})r_4^{ij} + [\theta_r^{ij} - 2 + 2\beta_{ij}]r_3^{ij}}{\theta_r^{ij}-1} \leq \omega_{ij}, & \beta_{ij}\in(\frac{1}{2}, \frac{3-\theta_r^{ij}}{4}] \\ \frac{[\theta_r^{ij} - 1 + 2\beta_{ij}]r_4^{ij} + (2-2\beta_{ij})r_3^{ij}}{1+\theta_r^{ij}} \leq \omega_{ij}, & \beta_{ij}\in(\frac{3-\theta_r^{ij}}{4}, 1] \end{cases} \quad (20)$$

where $\omega_{ij} = \sum_{b\in B}(pe_{ij}^b + pl_{ij}^b)$. The first two inequalities are derived from $\lambda = 0$, and the latter two from $\lambda = 1$.

Based on the above analysis, the equivalent deterministic model can be rewritten as the following:

$$\begin{aligned} \min \quad & T_{max} \\ \text{subject to} \quad & \pi_\beta^*(d;\theta) \leq \sum_{b\in B}(pe_{ij}^b + pl_{ij}^b), \\ & \qquad \forall(i,j)\in A,\ \beta_{ij}\in D_\beta^n, \\ & \text{Constraints } (1)-(12). \end{aligned} \quad (21)$$

Given the values of the mode parameters, β_{ij}, θ_l^{ij} and θ_r^{ij}, this model is a mixed-integer linear programming model, which can be solved by a commercial solver [32]. The next section contains a case study to illustrate an application of the model.

V. Case Study

In this section, we test the performance of the proposed approach using a real case based on Shanghai rail line 1, which consists of 28 stations and connects central business districts with high demands at different stations. The study of Rail Line 1 involves running both real and virtual data. In the following, the problem tackled in this case study is first described, and the related parameters are set. Then, computational results are presented and discussed under variations of the parameter settings.

A. Case description and data setting

We consider a rail disruption between the Xinzhuang and Xujiahui station along Shanghai Rail Line 1 at 11:20 am, on November 12^{th}, 2021. Figure 2 illustrates the diagram of Rail Line 1, which contains 6 disrupted stations from Waihuanlu to Shanghai Indoor Stadium (indexed as 1-6 respectively). The

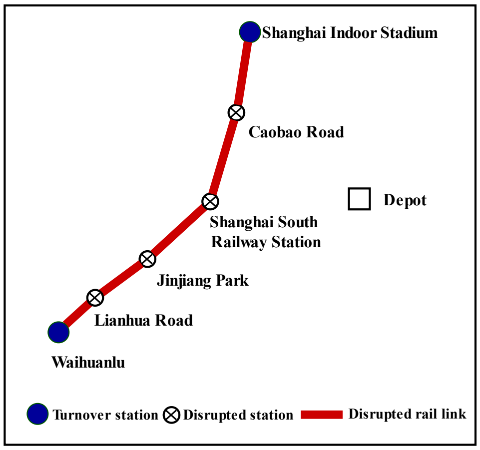

Fig. 2. Diagram of Shanghai Rail Line 1.

TABLE III
NUMBER OF BUSES AND DISPATCHING TIME FROM DEPOT TO STATIONS.

Number of buses	Dispatching time (min)				
	Station 1	Station 2	Station 3	Station 4	Station 5
15	16	19	11	9	12

TABLE IV
RUNNING TIME AND TRAPEZOIDAL PARAMETERS FOR OD PAIRS.

OD pair	Running time (min)	Trapezoidal parameters			
		r1	r2	r3	r4
(1,2)	3	396	443	489	536
(1,3)	7	173	194	214	235
(1,4)	13	116	129	143	156
(1,5)	19	97	108	120	131
(1,6)	24	139	156	172	189
(2,3)	4	748	836	924	1012
(2,4)	10	496	555	613	672
(2,5)	16	417	466	515	564
(2,6)	21	604	675	746	817
(3,4)	6	189	211	233	255
(3,5)	12	158	177	195	214
(3,6)	17	230	257	284	311
(4,5)	6	102	114	126	138
(4,6)	11	148	165	183	200
(5,6)	5	122	137	151	166

direction from Waihuanlu to Shanghai Indoor Stadium is defined as up direction and inversely down direction. Generally, the rail operators concerned simultaneously dispatch buses in both directions. For simplicity, we address the problem of optimizing the up direction trips.

Fifteen buses reserved in a surrounding depot are dispatched to provide the required bus bridging service. Bus capacity is 80 passengers each. Recall that there are 4 disrupted stations, then the number of all feasible routes is 15. The dispatching time from the depot to each disrupted station is shown in Table III. According to the daily passenger flow within each station of Shanghai Rail Line 1 in December 2018, we estimate the demand on the number of passengers who arrive at the relevant stations within half an hour after the disruption. The service time is assumed to 1 min at each boarding or alighting station.

Due to the noise in historical data and the impact of the disruption on passenger travel modes, precise information on passenger demand is unavailable. Let the quadruplet of trapezoidal parameters be expressed by $(r_1^{ij}, r_2^{ij}, r_3^{ij}, r_4^{ij})$. Based on the construction method of fuzzy membership function [33], [34], we derive the trapezoidal parameters estimated from passenger flow data as shown in Table IV. Note that the detailed procedures for generating the trapezoidal parameters based on given numeric data are summarized in Appendix B. Then we can represent the uncertain demand $[r_1^{ij}, r_2^{ij}, r_3^{ij}, r_4^{ij}; \theta_l^{ij}, \theta_r^{ij}]$, where θ_l^{ij} and θ_r^{ij} are determined by the decision maker based on the experts' experiences or subjective judgments. In this case study, the values of parameters θ_l and θ_r are specified as 0.24 and 0.15, respectively. Meanwhile, to ensure the evacuation work with high credibility levels, we denote the confidence level $\beta_{ij} = 0.9$. Comparisons are made between nominal and the proposed distributionally robust fuzzy models, where the nominal demand for each OD pair is equal to the mean value of the trapezoidal parameters.

B. Results analysis

The computational results are presented in this subsection, and the solution procedure is implemented using the CPLEX 12.10 commercial software. All the experiments are performed on a PC with AMD 1.80 GHz CPU and 4 GB RAM under Windows 8.

Table V shows the objective values and optimal solutions of the nominal and distributionally robust fuzzy models. The column named "Travel mode" indicates whether the bus bridging route mode is "express" or "local", with "E" denoting the express mode and "L" the local mode. Note that when the bridging route has only two adjacent stations, e.g., 1-2, we classify it as an express mode. For the nominal model, all 15 buses are dispatched to the disrupted stations. 8 buses are assigned to different local routes, and the rest are assigned to 5 different express routes. The maximum evacuation time is 147 min. For the distributionally robust fuzzy model, all 15 buses are also dispatched to the disrupted stations. 9 buses are assigned to 8 different local routes, and the rest are assigned to 4 different express routes. The maximum evacuation time is 169 min.

By comparing the objective values of these two models, we find that the distributionally robust fuzzy model is 22 min higher than the nominal model. This is because the distributionally robust fuzzy model considers the uncertainty of the demand, resulting in a more robust solution. Otherwise, the parametric settings for both models are the same. Suppose that the demands in each station take two extreme values (i.e., nominal value and maximum perturbation value). In this case, when sticking to the optimally assigned bridging routes, the travel frequency of the buses is increased accordingly. As a result, for the nominal model, the actual maximum evacuation time is 184 min, which is 25.17% larger than the objective value of 147 min derived from the optimal solution of the nominal situation. For the distributionally robust fuzzy model, the actual maximum evacuation time is 188 min, which is 16.77% larger than the objective value of 161 min derived by the optimal solution of the nominal situation. This 16.77% increase in distributionally robust value is less than the 25.17% increase of the actual value when the actual demand violates the nominal demand. The stability of emergency decision-making can effectively stabilize social order and reduce the loss of people's lives and property. Therefore, the distributionally robust fuzzy optimization method can provide a better uncertainty-immunized solution for the surging passenger flow emergency management problem.

C. Influence of parameter β_{ij}

We next identify the influence of the confidence level β_{ij} on the potential of achieving the optimal bridging decisions. For fixed parameters $\theta_l^\eta = 0.24$ and $\theta_r^\eta = 0.15$, the parameter β_{ij} changes with a step of 0.1 within the interval [0.7, 0.9]. The results are reported in Table VI. The second column lists the number of vehicles assigned in each disrupted station for two travel modes. The third column named "Mean value," represents the mean values of the distributionally robust model when demands take two extreme values with a probability 0.5 each. The last column, "Perturbation value," indicates the difference in the percentage of the values derived from the distributionally robust fuzzy model when there is a constraint violation. That is, the actual objective value when sticking to the optimal solution where the actual demand is against it.

From Table VI, with the decrease of the confidence level β, the mean value of the distributionally robust fuzzy model increases first and then decreases. In contrast, the fluctuation value decreases first and then increases. This means a higher robust price (mean value) can make the result more stable. With this policy, the government management department's decisions should balance the emergency plan's robustness and efficiency.

VI. Extensions

In this section, we extend our model by relaxing assumption 1 in Section 3 to allow the passengers to get on/off the bus at each bus station. The extended model with improved vehicle capability constraints is formulated as follows

$$\min \quad T_{max}$$

$$\text{s.t.} \quad T_e^b = \sum_{(i,j)\in A} (f_i + s_i + tb_{ij}) \times x_{ij}^b + \sum_{(i,j)\in A} (tb_{ij} + tb_{ji}) \times m_{ij}^b,\ \forall b \in B \tag{22}$$

$$T_l^b = \sum_{i\in S}(f_i + s_i) \times y_i^b + \sum_{i\in S} tb_{i,i+1} \times z_{i,i+1}^b + \sum_{i\in S}(tb_{i,i+1} + t_{i+1,i} + s_i \times y_i^b) \times n_{i,i+1}^b, \forall b \in B \tag{23}$$

$$n_{i'j'}^b \geq n_{i,i+1}^b - M*(1 - z_{i'j'}^b), \forall (i,j) \in A, b \in B, \quad i' = i, i+1, \cdots, j-1, j' = i'+1, i'+2, \cdots, j \tag{24}$$

$$n_{i'j'}^b \leq n_{i,i+1}^b + M*(1 - z_{i'j'}^b), \forall (i,j) \in A, b \in B, \quad i' = i, i+1, \cdots, j-1, j' = i'+1, i'+2, \cdots, j \tag{25}$$

$$z_{ij}^b \leq y_i^b + z_{i-1,i}^b,\ \forall i,j \in S, b \in B \tag{26}$$

$$z_{0,1}^b = 0,\ \forall b \in B \tag{27}$$

$$C \times (n_{i',i'+1}^b + z_{i',i'+1}^b) \geq \sum_{k=i}^{i'} \sum_{k'=i'+1}^{j} pl_{k,k'}^b, \quad \forall (i,j) \in A, i' = i, \cdots, j-1, b \in B \tag{28}$$

$$\text{Constraints} \quad (3)-(7), (11) \text{ and } (13).$$

TABLE VII
COMPARISON RESULTS OF BENCHMARK MODEL AND EXTENDED MODEL.

Model	Bus index	Travel mode	Route	Roundtrips	Time
BM	1	Local	TS1-DS1-DS2-DS3	6	108
	2	Express	DS1-DS3	3	38
EM	1	Local	TS1-DS1-DS2-DS3	4	76
	2	Express	TS1-DS2	4	48

The objective function is to minimize the clear evacuation time. Constraints (22) and (23) calculate the travel time on express and local routes, respectively, where tb_{ij} means the sum of inter-station travel time and service time at station j, i.e., $tb_{ij} = t_{ij} + s_j$. Constraints (24) and (25) ensure that bus b runs over the same roundtrips from station i to station j if it is dispatched to a local route. Constraint (26) imposes that bus b can serve the local route (i, j) only if it is dispatched to station i or the route $(i-1, i)$ is accessible. Constraint (27) is the initial condition. Constraint (13) certifies that the maximum capacity of each vehicle on a local route is not exceeded. Note that the total number of passengers on the bus at station k between routes (i, j) is equal to the sum of the number of passengers boarding from station i to station k minus the number of passengers alighting at station k. Constraints (3)-(7), (11) and (13) have not changed and are described before.

In order to examine the model's performance, the numerical experiment is based on the simple illustrative example in Section 3. Recall that a rail line service is interrupted in stations DS1, DS2 and DS3, and a bus line joining these stations will be used for bus bridging service. The parameters of dispatching time, inter-station running time and service time are the same in the simple example. To illustrate the performance of the extended model, we generate additional 20 passengers' demand from station DS1 to station DS3. The optimal operation strategies of the benchmark model (BM) and the extended model (EM) are shown in Table VII.

As shown in Table VII, in the BM, bus 1 is dispatched to station TS1 first (3 time units) and then transports passengers over the local route TS1-DS1-DS2-DS3 (9 time units). The roundtrip of this route is 6 and the corresponding travel time is 96 time units. Then, the total travel time of bus 1 is 108 time units. Bus 2 is dispatched from the depot to station DS1 first (3 time units), then transports passengers over the express route from station DS1 to DS3 (5 time units). The roundtrip of bus 2 is 3, and the corresponding travel time is 40 time units. Then, the total travel time of bus 2 is 48 time units. In the EM, bus 1 is also dispatched to station TS1 and travels over the local route TS1-DS1-DS2-DS3. The roundtrip of this route is 4, and the corresponding travel time is 64 time units. Then, the total travel time of bus 1 is 76 time units. For bus 2, it runs over the express route TS1-DS2 with 4 roundtrips, and the total travel time is 48 time units. The results show that the maximum evacuation time of EM is lower than that of BM.

However, if passengers can get on the bus at the intermediate stations, the service time will increase because the bus needs to turn back between rail stations and bus stations. Therefore, it is rational to consider an additional transfer time between the rail station and the bus station. We denote the transfer time as ts_i, and then the service time at the disrupted stations is

TABLE V
OBJECTIVE VALUES AND SOLUTIONS OF NOMINAL AND DISTRIBUTIONALLY ROBUST FUZZY MODELS.

Nominal model					Distributionally robust fuzzy model				
Bus index	Travel mode	Travel route	Roundtrips	Evacuation time (min)	Bus index	Travel mode	Travel route	Roundtrips	Evacuation time (min)
1	L	1-2	5		1	L	4-5-6	4	
2	L	1-2-3-4-5	1		2	L	3-4-5	5	
3	L	2-3	10		3	L	1-2-3-4-5	3	
4	L	3-4-5	4		4	L	2-3-4-5-6	2	
5	L	2-3-4-5	3		5	L	2-3-4-5-6	2	
6	L	1-2-3-4	3		6	L	1-2-3-4-5-6	2	
7	L	4-5-6	3		7	L	1-2-3	8	
8	L	3-4-5-6	3	147	8	L	2-3-4-5	3	169
9	E	5-6	2		9	L	2-3	13	
10	E	2-5	3		10	E	5-6	2	
11	E	2-4	5		11	E	2-6	2	
12	E	1-6	2		12	E	2-6	2	
13	E	2-6	2		13	E	2-6	2	
14	E	2-6	2		14	E	2-4	6	
15	E	2-6	2		15	E	3-6	3	

TABLE VI
PRICE AND VIOLATION VALUES OF DISTRIBUTIONALLY ROBUSTNESS WITH DIFFERENT CONFIDENCE LEVELS.

β	Number of buses										Mean value(min)	Perturbation value(percentage)
	Station 1		Station 2		Station 3		Station 4		Station 5			
	E	L	E	L	E	L	E	L	E	L		
0.9	0	3	4	4	1	1	0	1	1	0	174.5	16.77%
0.8	1	2	2	6	1	1	0	1	1	0	178.5	11.24%
0.7	0	3	3	5	0	2	0	1	1	0	169.5	20.13%

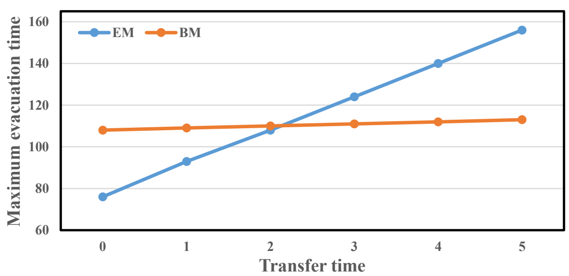

Fig. 3. Comparison between BM and EM on maximum evacuation time.

equal to the sum of boarding/alighting time and transfer time, i.e., s_i+ts_i. To describe the different transfer times between the rail station and the bus station, the parameter ts_i changes with a step of 1 within the interval $[1, 5]$. The comparison results are shown in Figure 3. One can find that with the increase in transfer time, the maximum evacuation time of EM increased significantly and finally exceeded the maximum evacuation time of BM. This indicates that EM applies to the scenario where the rail station is near the bus station, while BM is suitable for long-distance transfers.

VII. CONCLUSION

This paper has developed a new distributionally robust optimization approach for the bus bridging problem involving credibilistic constraints. In our proposed optimization model, only partial distribution information about the passenger demands in each station was assumed to be available, characterized by parametric interval-valued possibility distributions and their associated uncertainty distribution sets. Based on the equivalent representation of credibilistic constraints, we have converted the original optimization problem into equivalent robust counterpart models, which can be resolved by existing official solver software.

A real-world case study of Shanghai Rail Line 1 has been utilized to demonstrate the effectiveness of our distributionally robust optimization method. In contrast to previous studies, the experimental results have illustrated that the proposed approach can provide a better uncertainty-immunized solution. The optimal bus allocation, route selection and frequency determination solution obtained from our model jointly offer a robust strategy for effectively dealing with the uncertainty generated from the small perturbations around the nominal demand.

This study only considers problems involving single line bus bridging over routes with the affected line. It can be extended by introducing interline routes to connect with other rail lines. Another problem variant can be developed by considering multi-objective optimization models that simultaneously account for minimizing passenger delay time and total bridging time. In addition, other uncertain parameters, for example, travel time, may also be considered as the next step of this research.

Appendix A. Proof of Theorem 1

Since the possibility distribution of ζ_{ij} is a parametric interval-valued trapezoidal fuzzy variable [4] and ζ_{ij}^{λ} is a λ selection variable of ζ_{ij}, the parametric interval-valued possibility distribution $\mu(d;\theta,\lambda)$ of ζ_{ij} is

$$\mu(d;\theta,\lambda)=\begin{cases}\dfrac{[1+\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij}](\zeta_{ij}^{\lambda}-r_1^{ij})}{r_2^{ij}-r_1^{ij}}, & r_1^{ij}<\zeta_{ij}^{\lambda}\le\dfrac{r_1^{ij}+r_2^{ij}}{2}\\ \dfrac{[1-\lambda_{ij}\theta_r^{ij}+(1-\lambda_{ij})\theta_l^{ij}]\zeta_{ij}^{\lambda}+[\lambda_{ij}\theta_r^{ij}+(1-\lambda_{ij})\theta_l^{ij}]r_2^{ij}-r_1^{ij}}{r_2^{ij}-r_1^{ij}}, & \dfrac{r_1^{ij}+r_2^{ij}}{2}<\zeta_{ij}^{\lambda}\le r_2^{ij}\\ 1, & r_2^{ij}<\zeta_{ij}^{\lambda}\le r_3^{ij}\\ \dfrac{[\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij}-1]\zeta_{ij}^{\lambda}-[\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij}]r_3^{ij}+r_4^{ij}}{r_4^{ij}-r_3^{ij}}, & r_3^{ij}<\zeta_{ij}^{\lambda}\le\dfrac{r_3^{ij}+r_4^{ij}}{2}\\ \dfrac{[1+\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij}](r_4^{ij}-\zeta_{ij}^{\lambda})}{r_4^{ij}-r_3^{ij}}, & \dfrac{r_3^{ij}+r_4^{ij}}{2}<\zeta_{ij}^{\lambda}\le r_4^{ij}\end{cases}$$

According to the definition of credibility measure [3], the credibility constraints can be deal with in the following way.

If $\beta_{ij}<0.5$, then we have

$$\begin{aligned}&\mathrm{Cr}\left\{\zeta_{ij}^{\lambda}\le\omega_{ij}\right\}\\&=\frac{1}{2}\left\{1+\sup_{d\le u}\mu(d;\theta,\lambda)-\sup_{d>u}\mu(d;\theta,\lambda)\right\}\\&=\frac{1}{2}\sup_{d\le u}\mu(d;\theta,\lambda),\end{aligned}$$

where $\omega_{ij}=\sum_{b\in B}(pe_{ij}^{b}+pl_{ij}^{b})$. Thus, the credibility constraint $\mathrm{Cr}\left\{\omega_{ij}\ge\zeta_{ij}^{\lambda}\right\}\ge\beta_{ij}$ is equivalent to $\sup_{d\le u}\mu(d;\theta,\lambda)\ge 2\beta_{ij}$. Denote

$$\zeta_{\inf}(\theta)=\inf\left\{u\,\middle|\,\sup_{d\le u}\mu(d;\theta,\lambda)\ge\beta_{ij}\right\}$$

for $\beta_{ij}\in(0,1]$, then we have $\bar{d}_{\inf}(2\beta_{ij})\le u$. Note that when $d_{ij}=r_1^{ij}$, $\mu(r_1^{ij})=0$, and when $d_{ij}=\frac{r_1^{ij}+r_2^{ij}}{2}$, $\mu(\frac{r_1^{ij}+r_2^{ij}}{2})=\frac{(1+\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij})}{2}$.

If $0<2\beta_{ij}\le\frac{(1+\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij})}{2}$, i.e., $\beta_{ij}\in(0,\frac{(1+\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij})}{4}]$ then based on function (VII), the following equation is obtained:

$$\frac{[1+\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij}](\zeta_{ij}^{\lambda}-r_1^{ij})}{r_2^{ij}-r_1^{ij}}=2\beta_{ij}.$$

Solving the above equation, we have

$$\bar{d}_{\inf}(2\beta)=\frac{2\beta_{ij}r_2^{ij}+[\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij}+1-2\beta_{ij}]r_1^{ij}}{1+\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij}}.$$

Note that when $d_{ij}=r_2^{ij}$, $\mu(r_2^{ij})=1$.

If $\frac{(1+\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij})}{2}<2\beta_{ij}\le 1$, i.e., $\beta_{ij}\in(\frac{(1+\lambda_{ij}\theta_r^{ij}-(1-\lambda_{ij})\theta_l^{ij})}{4},\frac{1}{2}]$ then based on function (VII), the following equation is obtained:

$$\frac{[1-\lambda_{ij}\theta_r^{ij}+(1-\lambda_{ij})\theta_l^{ij}]d_{ij}+[\lambda_{ij}\theta_r^{ij}+(1-\lambda_{ij})\theta_l^{ij}]r_2^{ij}-r_1^{ij}}{r_2^{ij}-r_1^{ij}}=2\beta_{ij}.$$

Solving the above equation, we have

$$\bar{d}_{\inf}(2\beta)=\frac{[2\beta_{ij}-\lambda_{ij}\theta_r^{ij}+(1-\lambda_{ij})\theta_l^{ij}]r_2^{ij}+(1-2\beta_{ij})r_1^{ij}}{1-\lambda_{ij}\theta_r^{ij}+(1-\lambda_{ij})\theta_l^{ij}}.$$

The proof of assertions (i)-(ii) is complete. The other two asserting in theorem 1 can be proved similarly.

Appendix B. Generate trapezoidal parameters

The construction method of trapezoidal parameters is divided into two phases. First, according to the historical information of daily passenger flow at each station, we obtain a collection of numeric data $\Omega_{ij}=\{d_{ij}^1,d_{ij}^2,\cdots,d_{ij}^N\}$, which can be formalized by $\xi_{ij}=(r_1^{ij},r_2^{ij},r_3^{ij},r_4^{ij})$. Then, we calculate the mean value of Ω_{ij}, i.e., $\bar{d}_{ij}=\sum_{n=1}^{N}d_{ij}^n/N$. Second, we denote the membership degree of d_{ij}^n as $\mu(d_{ij}^n)$, which is computed by

$$\mu\left(d_{ij}^n\right)=\begin{cases}\frac{d_{ij}^n-r_1^{ij}}{r_2^{ij}-r_1^{ij}}, & \text{if } d_{ij}^n<r_2^{ij}\\ 1, & \text{if } r_2^{ij}\le d_{ij}^n<r_3^{ij}\\ \frac{r_2^{ij}-d_{ij}^n}{r_3^{ij}-r_2^{ij}}, & \text{if } r_3^{ij}\le d_{ij}^n\le r_4^{ij}.\end{cases}$$

Then, we solve the following maximization problem to determine r_1^{ij} and r_2^{ij}

$$\begin{aligned}\max\quad&\sum_{r_1^{ij}\le d_{ij}^n<r_2^{ij}}\mu(d_{ij}^n)\cdot\exp\left(-\alpha_{ij}|r_2^{ij}-r_1^{ij}|\right)+\\&\mathrm{crad}\{d_{ij}^n|d_{ij}^n\in[r_2^{ij},\bar{d}_{ij}]\}\cdot\exp\left(-\alpha_{ij}|\bar{d}_{ij}-r_2^{ij}|\right),\end{aligned}$$

where $\sum_{r_1^{ij}\le d_{ij}^n<r_2^{ij}}\mu(d_{ij}^n)$ means the objective of maximizing the number of covered demand data points for each OD pair, $\exp\left(-\alpha_{ij}|r_2^{ij}-r_1^{ij}|\right)$ is the objective of minimizing the support length $|r_2^{ij}-r_1^{ij}|$, α_{ij} is a positive parameter, and $\mathrm{crad}\{d_{ij}^n|d_{ij}^n\in[r_2^{ij},\bar{d}_{ij}]\}$ means the number of demand data points for each OD pair covered by $[r_2^{ij},\bar{d}_{ij}]$. Likewise, solve

the following maximization problem to determine r_3^{ij} and r_4^{ij}

$$\max \sum_{r_3^{ij} \le d_{ij}^n < r_4^{ij}} \mu(d_{ij}^n) \cdot \exp\left(-\alpha_{ij}|r_4^{ij} - r_3^{ij}|\right) + \mathrm{crad}\{d_{ij}^n | d_{ij}^n \in [\bar{d}_{ij}, r_3^{ij}]\} \cdot \exp\left(-\alpha_{ij}|\bar{d}_{ij} - r_3^{ij}|\right).$$

REFERENCES

[1] A. Shalaby, L. Li, and E. Diab, “Rail transit disruption management: A comprehensive review of strategies and approaches, *Handb. Public Transp. Res.*, pp. 280313, 2021.

[2] L. Kang and Q. Meng, “Two-phase decomposition method for the last train departure time choice in subway networks, *Transp. Res. Part B Methodol.*, vol. 104, pp. 568582, 2017.

[3] B. Liu and Y.-K. Liu, “Expected value of fuzzy variable and fuzzy expected value models, *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 4, pp. 445450, 2002.

[4] Y. Liu and Y.-K. Liu, “The lambda selections of parametric interval-valued fuzzy variables and their numerical characteristics, *Fuzzy Optim. Decis. Mak.*, vol. 15, no. 3, pp. 255279, 2016.

[5] Z.-Q. Liu and Y.-K. Liu, “Type-2 fuzzy variables and their arithmetic, *Soft Comput.*, vol. 14, no. 7, pp. 729747, 2010.

[6] Z. Guo and Y. Liu, “Modelling single-period inventory problem by distributionally robust fuzzy optimization method, *J. Intell. Fuzzy Syst.*, vol. 35, no. 1, pp. 10071019, 2018.

[7] L. Kang, X. Zhu, H. Sun, J. Wu, Z. Gao, and B. Hu, “Last train timetabling optimization and bus bridging service management in urban railway transit networks, *Omega*, vol. 84, pp. 3144, 2019.

[8] J. G. Jin, K. M. Teo, and A. R. Odoni, “Optimizing bus bridging services in response to disruptions of urban transit rail networks, *Transp. Sci.*, vol. 50, no. 3, pp. 790804, 2016.

[9] Y. Chen and K. An, “Integrated optimization of bus bridging routes and timetables for rail disruptions, *Eur. J. Oper. Res.*, vol. 295, no. 2, pp. 484498, 2021.

[10] E. Van der Hurk, H. N. Koutsopoulos, N. Wilson, L. G. Kroon, and G. Marti, “Shuttle planning for link closures in urban public transport networks, *Transp. Sci.*, vol. 50, no. 3, pp. 947965, 2016.

[11] Y. Wang, X. Yan, Y. Zhou, and W. Zhang, “A feeder-bus dispatch planning model for emergency evacuation in urban rail transit corridors, *PLoS One*, vol. 11, no. 9, p. e0161644, 2016.

[12] H. Hu, Y. Gao, J. Yu, Z. Liu, and X. Li, “Planning bus bridging evacuation during rail transit operation disruption, *J. Urban Plan. Dev.*, vol. 142, no. 4, p. 4016015, 2016.

[13] H. Yin, J. Wu, H. Sun, Y. Qu, X. Yang, and B. Wang, “Optimal bus-bridging service under a metro station disruption, *J. Adv. Transp.*, vol. 2018, 2018.

[14] W. Gu, J. Yu, Y. Ji, Y. Zheng, and H. M. Zhang, “Plan-based flexible bus bridging operation strategy, *Transp. Res. Part C Emerg. Technol.*, vol. 91, pp. 209229, 2018.

[15] C. Luo and L. Xu, “Railway disruption management: Designing bus bridging services under uncertainty, *Comput. Oper. Res.*, vol. 131, p. 105284, 2021.

[16] M. B. Kar, B. Roy, S. Kar, S. Majumder, and D. Pamucar, “Type-2 multi-fuzzy sets and their applications in decision making, *Symmetry*, vol. 11, no. 2, p. 170, 2019.

[17] S. Singh and H. Garg, “Symmetric triangular interval type-2 intuitionistic fuzzy sets with their applications in multi criteria decision making, *Symmetry* , vol. 10, no. 9, p. 401, 2018.

[18] Y. Liu and Y.-K. Liu, “Distributionally robust fuzzy project portfolio optimization problem with interactive returns, *Appl. Soft Comput.*, vol. 56, pp. 655668, 2017.

[19] M. Deveci, V. Simic, S. Karagoz, and J. Antucheviciene, “An interval type-2 fuzzy sets based Delphi approach to evaluate site selection indicators of sustainable vehicle shredding facilities, *Appl. Soft Comput.*, vol. 118, p. 108465, 2022.

[20] Y. Liu, R. M. Rodriguez, H. Hagras, H. Liu, K. Qin, and L. Martinez, “Type-2 fuzzy envelope of hesitant fuzzy linguistic term set: A new representation model of comparative linguistic expression, *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 12, pp. 23122326, 2019.

[21] S. K. Das, S. K. Roy, and G.-W. Weber, “Application of type-2 fuzzy logic to a multiobjective green solid transportationlocation problem with dwell time under carbon tax, cap, and offset policy: Fuzzy versus nonfuzzy techniques, *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 11, pp. 27112725, 2020.

[22] A. Ben-Tal and A. Nemirovski, *Lectures on modern convex optimization: Analysis, algorithms, and engineering applications*. SIAM, 2001.

[23] A. Ben-Tal, L. El Ghaoui, and A. Nemirovski, *Robust optimization*. Princeton university press, 2009.

[24] J. Liang, J. Wu, Y. Qu, H. Yin, X. Qu, and Z. Gao, “Robust bus bridging service design under rail transit system disruptions, *Transp. Res. Part E Logist. Transp. Rev.*, vol. 132, pp. 97116, 2019.

[25] A. Kulshrestha, Y. Lou, and Y. Yin, “Pickup locations and bus allocation for transitbased evacuation planning with demand uncertainty, *J. Adv. Transp.*, vol. 48, no. 7, pp. 721733, 2014.

[26] V S. Shahparvari and B. Abbasi, “Robust stochastic vehicle routing and scheduling for bushfire emergency evacuation: An Australian case study, *Transp. Res. Part A Policy Pract.*, vol. 104, pp. 3249, 2017.

[27] M. Tavana, M. Toloo, N. Aghayi, and A. Arabmaldar, “A robust cross-efficiency data envelopment analysis model with undesirable outputs, *Expert Syst. Appl.*, vol. 167, p. 114117, 2021.

[28] Y. Lv, X. D. Yan, W. Sun, and Z. Y. Gao, “A risk-based method for planning of bussubway corridor evacuation under hybrid uncertainties, *Reliab. Eng. Syst. Saf.*, vol. 139, pp. 188199, 2015.

[29] J. Halliwell and Q. Shen, “Linguistic probabilities: Theory and application, *Soft Comput.*, vol. 13, no. 2, pp. 169183, 2009.

[30] A. Ben-Tal, B. Do Chung, S. R. Mandala, and T. Yao, “Robust optimization for emergency logistics planning: Risk mitigation in humanitarian relief supply chains, *Transp. Res. part B Methodol.*, vol. 45, no. 8, pp. 11771189, 2011.

[31] M. Yang, Y. Liu, and G. Yang, “Multi-period dynamic distributionally robust pre-positioning of emergency supplies under demand uncertainty, *Appl. Math. Model.*, vol. 89, pp. 14331458, 2021.

[32] X. Bai, X. Li, R. Jia, and Y. Liu, “A distributionally robust credibilistic optimization method for the economic-environmental-energy-social sustainability problem, *Inf. Sci.* , vol. 501, pp. 118, 2019.

[33] W. Pedrycz, and M. Song, “Granular fuzzy models: A study in knowledge management in fuzzy modeling,” *Int. J. Approx. Reason.*, vol. 53, no. 7, pp. 1061-1079, 2012.

[34] J. Zhou, X. Li, S. Kar, G. Zhang, and H. Yu, “Time consistent fuzzy multi-period rolling portfolio optimization with adaptive risk aversion factor,” *J. Amb. Intel. Hum. Comp.*, vol. 8, no. 5, pp. 651666, 2017.