Mathematical and numerical modeling of hydraulic fractures for non-Newtonian fluids

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The aim of this thesis is to construct an accurate and effective numerical algorithm to solve a problem of hydraulic fracturing for non--Newtonian fluids of power--law type rheology. Also, to improve the existing semi--analytical solutions using the recent advances in the area.

The theoretical background is presented, along with the practical applications and physical processes driving the fracture growth. Equations used to mathematically formulate the problem are derived. A full mathematical description is supplemented by boundary and initial conditions. Basic 1D models are presented for various crack propagation regimes.

Mechanisms used extensively throughout this work are thoroughly analysed, amongst which are: (i) numerical integration, (ii) regularisation techniques, (iii) implementation of boundary conditions, (iv) utilisation of appropriate dependent variables, (v) asymptotic behaviour of solution, (vi) appropriate meshing strategy. Each of these methods is described in detail and investigated.

A universal particle velocity based algorithm for simulating hydraulic fractures is proposed. The computations are based on two dependent variables: the crack opening and the reduced particle velocity. The application of the latter facilitates utilisation of the local condition of Stefan type (speed equation) to trace the fracture front. The condition is given in a general explicit form which relates the crack propagation speed (and the crack length) to the solution tip asymptotics.

A number of analytical benchmark solutions are derived. They are employed to validate the computational accuracy of the proposed algorithm. Moreover, the performance of the numerical scheme is tested against other solutions available in the literature.

Following the analysis of the performance of the algorithm, new improved approximations are provided for each model. They can be used as benchmark solutions for testing other algorithms.

The extensive computations prove that the numerical scheme is stable and efficient. It provides accurate results for various hydraulic fracturing models and regimes, as well as fracturing fluids. The algorithm works equally well for time--independent and transient versions of the problem. The utilisation of a modular structure and the adaptive character of its basic blocks result in a flexible numerical scheme.

The numerical code has been provided as a supplementary attachment to the electronic version of this thesis, and is also available on request from the author.