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Impacts of trade credit on pricing decisions of complementary products

Da Ren\textsuperscript{a}, Yanfei Lan\textsuperscript{a,b,*}, Changjing Shang\textsuperscript{b}, Jiahe Wang\textsuperscript{a}, Chenlin Xue\textsuperscript{a}

\textsuperscript{a} College of Management & Economics, Tianjin University, Tianjin 300072, China,
\textsuperscript{b} Department of Computer Science, Aberystwyth University, Aberystwyth SY23 3DB, UK,
dren@tju.edu.cn, lanyf@tju.edu.cn, cns@aber.ac.uk, wangjiahe0326@hotmail.com,
xuechenlin@hotmail.com

Abstract

To explore the impacts of trade credit on the pricing decisions of complementary product manufacturers, we establish a Bertrand model of two-echelon supply chains. Such a supply chain consists of two duopolistic suppliers that provide complementary products to a monopolistic retailer in three scenarios: 1) no supplier extends trade credit, 2) only one supplier extends trade credit, and 3) both suppliers extend trade credit. We find that the impacts of trade credit on the profit of each supply chain member are dependent on the difference in the opportunity cost between the upstream suppliers that extend trade credit and the downstream retailer. If the value is negative, one supplier will increase its profit by extending trade credit, thereby enhancing the profits of the other supplier and the retailer simultaneously. Further, if the value is negative, both suppliers adopting trade credit will provide more benefits to the whole supply than only single supplier adopting it. On the contrary, if the value is positive, no party extending trade credit will benefit all participants the most.

Key words: supply chain management, production modeling, trade credit, complementary product, pricing decision

1. Introduction

Contrary to the concept of cash on delivery, which means that a buyer pays the supplier upon delivery of the goods, trade credit refers to the practice that the supplier credits the buyer a term within which to delay the payment. Although the practice of trade credit may seem costly for the suppliers (i.e., it may commit their capital otherwise useable for other businesses and exaggerate their accounts receivable), it is the largest source of external financing for most business-to-business firms in the United States, especially for startup and growing firms (Chang and Rhee, 2011). Lee and Stowe (1993) calculated the amount of trade credit in 1985 and concluded that lending from suppliers far exceeded the amount of loans from the entire banking system. Rajan and Zingales (1998) reported that accounts receivable and accounts payable comprised 15 and 17.8 percent of the assets in the United States, respectively.

However, although it has been widely adopted, trade credit is nevertheless difficult to
achieve in certain instances. Suppliers and retailers have different assets and operating capabilities in different industries in practice. Among many differences between suppliers and retailers, it is particularly common for them to have different opportunity costs, which would directly lead to different capital borrowing costs. Under such conditions, in order to further improve the efficiency of the supply chain, when suppliers and retailers reach a trade credit agreement, they should consider the opportunity cost of trade credit, which might generate a series of problems and hence change the attitudes of suppliers and retailers towards trade credit.

A primary problem might arise in this context is that how the difference in opportunity cost between suppliers and retailers would affect the implementation of trade credit. If the supplier provides trade credit to the retailer, the account receivables would increase correspondingly, which would incur borrowing costs; otherwise, the supply chain efficiency would not promote because retailers, especially small and medium-size enterprises (SMEs), lack corresponding financial support. For the retailer, accessing to trade credit might further strengthen the desire to order more products, which might increase the efficiency of the entire supply chain. But the retailer also needs to consider the interest paying to the supplier and the profit if receives a trade credit. Therefore, it is difficult to directly judge whether suppliers or retailers prefer trade credit, which further increases difficulty in determining whether to adopt a trade credit policy. Besides, trade credit may not only have an impact on suppliers and retailers, but may also have an impact on their peers, especially suppliers that produce supplementary products. As for the parties in a supply chain, the pricing decisions of products might be the first and foremost question they need to consider, which could directly affect the benefits of all parties and efficiency of the entire supply chain.

However, most studies address the issue that how trade credit influences the relationship between suppliers and buyers (Peura et al., 2015). Only a few researchers have investigated the horizontal role of trade credit and recognized its importance for reducing competition. Particular findings include that monopoly power is negatively correlated with credit provision (Fisman and Raturi, 2003), whilst financially stronger suppliers may be able to exclude weak competitors from the market through a trade credit policy (Peura et al., 2015). However, no study has thus far analyzed the horizontal role of trade credit on complementary products, which emerges when customers have to buy more than one product at the same time to ensure full utility (Yue et al., 2006). For example, pencils and erasers, shuttlecocks and badminton rackets, and cars and petrol are all complementary products. In contrast to the fact that suppliers that manufacture substitutable products are competitors and may lose profits from each other’s sales, firms benefit from increases in the sales of complementary products.

Whether firms can raise the equilibrium price and profits over a certain supply chain by offering complementary products through extending trade credit remains unknown. Based on the above considerations, such interactions of trade credit between parties in supply chain producing and selling complementary products give rise to some interesting research questions to explore. First, what is the horizontal role of trade credit in the complementary product supply chain? Second, what is the mechanism by which trade credit affects pricing decisions? Third, how should a complementary product supply chain choose the trade credit policy to improve the overall efficiency of the supply chain?

To investigate the role of trade credit in a supply chain that sells complementary products,
we analyze a monopoly market consisting of two duopolistic suppliers that manufacture complementary products and sell their products to a monopolistic retailer. Both suppliers have the choice of extending trade credit to the retailer, although this action exposes them to an opportunity cost. In particular, we analyze three scenarios in which the two duopolistic suppliers make different decisions on whether to extend trade credit to the monopolistic retailer: (i) no supplier provides trade credit to the retailer, (ii) only one supplier extends trade credit to the retailer, and (iii) both suppliers provide trade credit to the retailer. By following the game-theoretical approach, the equilibrium solutions for the asymmetric setting in each scenario are obtained and compared. Then, by comparing the profits in these three scenarios under a symmetric structure, we discover novel insights into the potential impacts of trade credit upon the supply chain and the circumstances under which trade credit is valuable.

Our contributions to knowledge on this topic are summarized below.

First, regarding the equilibrium solutions under the asymmetric structure, we identify the optimal wholesale prices and retail prices in each scenario, showing that the impact of the trade credit policy on these optimal solutions is influenced by both (i) the difference between the interest rate that the retailer pays the suppliers and the retailer’s opportunity cost and (ii) the adjusted difference in the opportunity cost between the upstream suppliers that extend trade credit and the downstream retailer. When the interest rate is lower than the retailer’s opportunity cost, extending trade credit raises the wholesale price of the supplier that extends trade credit. Moreover, when the value of the latter factor is positive, trade credit has a positive effect on the wholesale price’s sensitivity to the cost of the supplier that chooses to provide trade credit, while it negatively affects the wholesale price’s sensitivity to the cost of the other supplier.

Second, with regard to the profits of the two duopolistic suppliers and the monopolistic retailer in the symmetric setting, we unearth the following fact: Whether the trade credit policy is beneficial to each individual in a supply chain is related to the value of the adjusted difference in the opportunity cost between the supplier that extends trade credit and the retailer. When the value is positive, no party extending trade credit is the most profitable strategy for all members, while when the value is negative, trade credit can help increase the profit of all supply chain members and specifically, both suppliers adopting the trade credit policy simultaneously benefits each one the most.

Third, through numerical examples, we illustrate our findings and demonstrate that both suppliers should always adopt a similar trade credit policy. In particular, when the adjusted opportunity cost difference is negative, trade credit can raise the efficiency of the entire supply chain, and both suppliers adopting a trade credit policy can maximize the profit of their own as well as the whole supply chain. However, when the adjusted opportunity cost difference is positive, no party will benefit from adopting the trade credit policy.

The rest of the paper is organized as follows. Section 2 presents the literature review. Section 3 describes the main problem addressed and the key notations to be utilized. Section 4 analyses the model and details the equilibrium solutions. Section 5 compares different scenarios with and without trade credit and investigates their impacts upon the profits of the supply chain members. Numerical examples are provided in Section 6. Section 7 presents several extensions. Section 8 concludes this paper and discusses the future research directions. Appendix A summarizes the notations and Appendix B presents the proofs of the propositions.
and theorems.

2. Literature review

2.1 Literature on trade credit

In recent years, trade credit as a type of loan provided by suppliers to their buyers and a commonly used provision in supply chain contracts has attracted considerable theoretical and practical research attention. On the aggregate balance sheets of non-financial U.S. businesses, for instance, accounts payable are three times as large as bank loans and 15 times as large as commercial papers (Barrot, 2016). This is despite the fact that extending trade credit exposes the creditor to larger default risks from their customers. This has initiated a long-standing discussion about the rationale behind trade credit. Early researchers pointed out that trade credit is mostly used by large suppliers, extending to their small customers to relieve downstream financial constraints. For examples, small firms use trade credit because it is hard for them to obtain credit from conventional financial institutions and suppliers lend to their customers to gain information about buyers cheaply and forge long-term relations (Schwartz, 1974; Petersen and Rajan, 1997). Biais and Gollier (1997) found that suppliers would hold more private information about their customers than external financial intermediaries would, thereby reducing their lending costs. However, studies such as Ng et al. (1999) and Klapper et al. (2012) concluded that small suppliers would use trade credit to assure buyers of their product quality.

In recent years, researchers have examined the roles of trade credit from more specific aspects. Ferrando and Mulier (2013) have empirically shown the impact of trade credit on financing growth and found that firms would rely more on trade credit to finance growth if they were more vulnerable to shocks in the financial market and more likely to be financially constrained. Zhong and Zhou (2013) proposed a model under which the presented trade credit policy could increase the profitability of each member as well as the whole two-echelon supply chain. Jacobson and Schedvin (2015) quantified the importance of trade credit chains for the propagation of corporate bankruptcies and found that a customer’s failure to pay would expose the creditor to a larger risk of bankruptcy. Fabbri and Klapper (2009) pointed out that firms would be more likely to extend trade credit to their customers if they received credit from their own suppliers, with the strong competition in the product market encouraging firms to match maturity between their accounts payable and accounts receivable. The same authors have also investigated the relation between suppliers’ bargaining power and the supply of trade credit, demonstrating that suppliers with weak bargaining power compared with their customers would be more likely to extend trade credit as a competitive tool in the product market (Fabbri and Klapper, 2016).

All these studies focus on the vertical roles of trade credit, however, little research has investigated the horizontal roles of trade credit in a supply chain. Our research is thus distinct from prior studies, as we analyze a supply chain including two duopolistic suppliers and examine how each supplier’s trade credit strategy affects the equilibrium solutions of both suppliers.
The literature on pricing decisions of complementary products generally reflects various perspectives. Some research investigates the strategy of bundling complementary products. For example, Venkatesh and Kamakura (2003) addressed how pricing strategies for complements or substitutes as a bundle differ from those for independently valued products. Yan and Bandyopadhyay (2011) developed a profit maximization model to explore the optimal bundling and pricing strategies, finding that the value of a bundling strategy would increase along with the size of the market and price sensitivity. Zhang and Wang (2018) used a linear demand model estimating the demand of manufacturers and retailers to examine the impact of fairness concern on the three-party supply chain coordination. Li et al. (2017) considered a dual channel supply chain to introduce a retailer’s risk-averse behavior into the manufacturer’s encroachment problem under asymmetric information, focusing specifically on how the factors might influence the optimal pricing decisions.

By forming a simultaneously played Bertrand model, Yue et al. (2006) investigated the optimal strategies for complementary firms to make pricing decisions under the assumption of information asymmetry, propounding that firms would obtain the highest profits if they form a strategic forecast information alliance and act collectively as an integrated single firm. Bhaskaran and Gilbert (2005) discovered that when a complementary product exists, a durable goods manufacturer would tend to increase the leasing prices of its product. Wei et al. (2013) proposed five pricing models of a supply chain with two complementary products manufacturers under decentralized decision-making and showed the pricing decisions with different market power structures among channel members. Using similar approaches, Wei et al. (2014) also analyzed decision on the pricing and warranty period of two complementary products by considering two manufacturers’ cooperation/non-cooperation strategies and also, the firms’ different bargaining powers. Zhao et al. (2016) studied the pricing decision of two complementary products in a two-level fuzzy supply chain with two manufacturers and one common retailer.

Compared with the aforementioned existing studies, the main difference of our work is that we concentrate on the effects of trade credit regarding the pricing decisions in a complementary product supply chain. To the best of our knowledge, this problem has not been appropriately addressed before.

In summary, this paper complements the existing supply chain finance literature in three ways. First of all, compared to previous studies on the relationship between upstream and downstream members of the supply chain by trade credit, we will focus on the impact of trade credit on peers, specifically the suppliers of producing complementary products. Secondly, we study the impact of trade credit on the pricing decision of complementary products from a new perspective, that has not been effectively addressed yet. Finally, we not only study the trade credit provided by only one party in the supply chain, but also conduct a comprehensive and comparative study of all the parties that may provide trade credit. Table 1 summarizes the existing related literature and points out that this article supplements the existing literature gaps.
Table 1: Summary of the related literature and new contributions

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3. Problem formulation and notations

Consider a supply chain consisting of two large duopolistic suppliers (labelled supplier 1 and supplier 2) and a small monopolistic retailer. The market is controlled by the two suppliers. Supplier $i$ provides product $i$ to the retailer at the wholesale price of $w_i$ and the unit manufacturing cost of product $i$ is $c_i$, and the retailer sells product $i$ to the consumer at
the retail price of $p_i$, where $i = 1, 2$. In the market, the two suppliers charge the wholesale
price first and then the retailer decides the retail prices for the two products based on those
 wholesale prices. Both suppliers may choose to provide trade credit to the retailer, even
though extending trade credit incurs an opportunity cost $I_i$, where $i = 1, 2$ and $r$, before they
receive payment for the products. Trade credit is a kind of loan provided by the supplier,
supplier receives interest from the retailer at the interest rate $\eta_i$, where $i = 1, 2$. The products
that the two suppliers provide to the retailer are complementary (i.e., consumers can obtain
the full utility only if they possess both products simultaneously).

The consumers in the market can therefore, be divided into three segments: Segment 1
only buys product 1, Segment 2 needs products 1 and 2, and Segment 3 buys product 2 only.
In this model, we assume that both suppliers and the retailer are risk neutral and that there is
no price discrimination among these three segments of consumers (Choi, 1996;
 Mukhopadhyay et al., 2011; Wu et al., 2012). We also assume that market demand for both
products is a linear function of the two retail prices (Zhu et al., 2016; Ma et al., 2018).

Then, demand for product 1 of Segment 1 is
$$D_{11}(p_1) = a_1 - \beta_1 p_1.$$  (1)
Similarly, demand for product 2 of Segment 3 is
$$D_{23}(p_2) = a_2 - \beta_2 p_2.$$  (2)
The demands for products 1 and 2 of Segment 2 are respectively:
$$D_{12}(p_1) = a - \beta_{11} p_1 - \beta_{12} p_2,$$  (3)
and
$$D_{22}(p_2) = a - \beta_{22} p_2 - \beta_{21} p_1.$$  (4)

In the above, we assume that the parameter $a_1$ is the primary demand for product 1
from Segment 1 and $a_2$ is that for product 2 from Segment 3; and that the parameter $a$
refers to the primary demand for products 1 and 2 from Segment 2. The parameters $\beta_1$, $\beta_2$,
$\beta_{11}$, and $\beta_{22}$ refer to the price sensitivities of a product’s demand to its own price, whilst
the parameters $\beta_{12}$ and $\beta_{21}$ denote the cross-price sensitivities of a product’s demand to the
price of its complementary product. We also assume that the parameters are all non-negative
as well as that $\beta_{11}$ is larger than $\beta_{12}$ and that $\beta_{22}$ is larger than $\beta_{21}$. In other words,
cross-price sensitivities are lower than self-price sensitivities (Wei et al., 2013; Shavandi et al.,
2015). This assumption suggests that the demand for the product is more sensitive to changes
in its own price than to changes in the price of its complementary product. Therefore, the total
demand for product 1 is the sum of demands for product 1 from Segments 1 and 2:
$$q_1 = (a_1 + a - (\beta_1 + \beta_{11}) p_1 - \beta_{12} p_2),$$  (5)
and the total demand for product 2 is the sum of demands for product 2 from Segments 2 and
3:
$$q_2 = (a_2 + a - (\beta_2 + \beta_{22}) p_2 - \beta_{21} p_1).$$  (6)

When supplier $i$ asks the retailer to pay on delivery, the profit of supplier $i$ equals the
product of unit profit and demand. Meanwhile, both suppliers can save an additional
opportunity cost, that is, $I_i(w_i - c_i)q_i, i = 1, 2$ (Wu et al., 2017). Therefore, the profit of
supplier $i$ that does not extend trade credit can be written as
$$\pi_i = (w_i - c_i)q_i + I_i(w_i - c_i)q_i, i = 1, 2.$$  (7)
By contrast, by providing trade credit, supplier $j$ ($j = 1, 2$) is not paid immediately upon
delivery of the products. Extending trade credit also generates an additional opportunity cost
burden for supplier \( j \), and the amount of this burden is related to the total manufacturing cost of the product, that is, \( I_j c_j q_j, j = 1,2 \) (Wu et al., 2017). Therefore, the profit of supplier \( j \) can be given as

\[
\pi_j = \left( 1 + r_j \right) w_j - c_j - I_j c_j q_j, j = 1,2.
\]  

(8)

When no party extends trade credit, the retailer’s profit is

\[
\pi_r = (p_1 - w_1) q_1 + (p_2 - w_2) q_2.
\]  

(9)

However, accepting trade credit not only generates interest, but also saves an additional opportunity cost, both of which are related to the total payment. Without loss of generality, we thus assume that when only one supplier extends trade credit to the retailer, that party is supplier 1. Then, the profit of the retailer is

\[
\pi_r = [p_1 - (1 + r_1 - I_r) w_1] q_1 + [p_2 - (1 + r_2 - I_r) w_2] q_2,
\]  

(10)

where \( I_r \) denotes the retailer’s opportunity cost.

As the market is controlled by the suppliers and the firms aim to maximize their profits, the Bertrand model can be reformulated as

\[
\max_{w_1} \pi_1(w_1, p_1^*, p_2^*) \max_{w_2} \pi_2(w_2, p_1^*, p_2^*) \max_{(p_1, p_2)} \pi_r(p_1, p_2)
\]  

(12)

4. Optimal solutions

In this section, we discuss and contrast the optimal wholesale prices and retail prices in the three examined scenarios: 1) no supplier extends trade credit to the retailer (Case N), 2) only one supplier extends trade credit to the retailer (Case S), and 3) both suppliers extend trade credit (Case D). In each scenario, the two suppliers move simultaneously to decide their wholesale prices and then the retailer decides the retail prices of the complementary products. The goal of both suppliers and the retailer is to maximize their own profits.

The game-theoretical approach is herein used to obtain the optimal wholesale prices and retail prices. In particular, the problem is solved by backwards inductions.

4.1 No supplier extending trade credit (Case N)

As the two duopolistic suppliers dominate the market, they announce their wholesale first and then the retailer decides on the price to charge for the two products. In this case, the equilibrium wholesale prices and retail prices are retained.

**Proposition 1.** When no supplier extends trade credit, the duopolistic suppliers’ wholesale prices \( w_1^N, w_2^N \) and retail prices \( p_1^N, p_2^N \) are given as

\[
w_1^N = \frac{B_1 B_2 - B_3 B_4}{B_2 B_5 - B_3 B_6},
\]  

(13)

\[
w_2^N = \frac{B_2 B_4 - B_1 B_6}{B_2 B_5 - B_3 B_6},
\]  

(14)

\[
p_1^N = A_1 + A_2 w_1^N + A_3 w_2^N,
\]  

(15)
\[ p_2^N = A_4 + A_5 w_2^N + A_6 w_1^N. \]  

where \( A_1, A_2, \ldots, A_6, B_1, B_2, \ldots, B_6 \) are constants, as defined in Appendix A.

Proposition 1 presents the same equilibrium solutions as the MS-Bertrand model in Wei et al. (2013), although the previous work did not consider a trade credit policy or the opportunity cost that suppliers may face. This is due to the assumption that the opportunity cost is linearly correlated to suppliers’ profit from the trade.

4.2 Only one supplier extending trade credit (Case S)

In this scenario, only one supplier provides trade credit to the monopolistic retailer. Without losing generality, we assume that the supplier that extends trade credit is supplier 1. Similar to Proposition 1, the two duopolistic suppliers and retailer make their decisions on the wholesale prices and retail prices sequentially based on the goal of profit maximization. The results are presented in Proposition 2.

**Proposition 2.** When only one supplier extends trade credit, the duopolistic suppliers’ equilibrium wholesale prices \( w_1^S, w_2^S \) and equilibrium retail prices \( p_1^S, p_2^S \) are given as

\[ w_1^S = \frac{2(1+r_1)(B_1 B_5 - B_2 B_3) + c_1 B_2 B_5 K_1}{2(1+r_1)(1+r_1-I_r) (B_2 B_5 - B_3 B_6)}, \quad (17) \]

\[ w_2^S = \frac{2(1+r_1)(B_2 B_6 - B_3 B_5) - c_1 B_2 B_6 K_1}{2(1+r_1)(B_2 B_5 - B_3 B_6)}, \quad (18) \]

\[ p_1^S = A_1 + (1 + r_1 - I_r) A_2 w_1^S + A_3 w_2^S, \quad (19) \]

\[ p_2^S = A_4 + A_5 w_2^S + (1 + r_1 - I_r) A_6 w_1^S, \quad (20) \]

where \( A_1, A_2, \ldots, A_6, B_1, B_2, \ldots, B_6, K_1 \) are constants, as defined in Appendix A.

As \( B_1, B_2, \ldots, B_6 \) are positive constants, intuitively, from Proposition 2, we know that the impact of the trade credit policy on product 1’s wholesale price is dependent on the value of \( K_1 \), which is the adjusted difference in the opportunity cost between the upstream supplier providing trade credit and the downstream retailer and also, on the difference between the interest rate \( r_1 \) and the retailer’s opportunity cost \( I_r \), while the wholesale price of product 2 is only influenced by \( K_1 \). Interestingly, we also find that in this case, the extent that trade credit affects the optimal solutions for both suppliers is only influenced by supplier 1’s unit manufacturing cost. Specifically, the higher the unit manufacturing cost of supplier 1, the more supplier 1’s trade credit policy affects the wholesale prices of the two products.

To shed light on the effects of the unilateral trade credit policy, after the optimal wholesale and retail prices of the two complementary goods in Case S are obtained, we compare the results in these two cases. This results in Corollary 1 below.

**Corollary 1.** The supplier unilaterally adopting a trade credit policy affects the wholesale prices of both products as follows:
Corollary 1 shows that when supplier 1 provides trade credit, the impact on the wholesale prices of the two products can run in different directions, owing to the impact of various parameters that are case-related. Intuitively, the effect on the wholesale price of supplier 1 originates from two parts: (i) the relation between the interest rate that the retailer pays and the retailer’s opportunity cost, which affects supplier 1’s wholesale price to a greater extent, and (ii) $K_1$, the adjusted difference in the opportunity cost between the upstream suppliers that extend trade credit and the downstream retailer, which affects the wholesale price’s sensitivity on cost.

From the proof of Proposition 2 (see Appendix B), we know that when supplier 1 extends trade credit, the impact on the retail price of product 1 from the wholesale price is $(1 + r_1 - I_r)$ times that in Case N. When $r_1 < I_r$, the retailer can therefore save more opportunity cost than the amount used to compensate the supplier, thus raising the retailer’s profit. Hence, the supplier who lends to the retailer is encouraged to charge a higher wholesale price to share the profit. However, $c_1 > \frac{2(1+r_1)(r_1-I_r)(B_1B_5-B_3B_4)}{B_2B_5K_1}$ which means that $w_1^S$ is smaller than $w_1^N$. Although supplier 1 wants to share the profit from the retailer by charging a higher wholesale price, this profit is not sufficient to compensate for the losses incurred by the reduced demand, and supplier 1 must therefore charge a lower wholesale price to generate a profit. By contrast, when $r_1 > I_r$, the retailer’s profit decreases to some extent, thereby reducing the amount that retailer is willing to pay the supplier, this leads to the equilibrium wholesale price dropping. However, $c_1 > \frac{2(1+r_1)(r_1-I_r)(B_1B_5-B_3B_4)}{B_2B_5K_1}$ which implies that $w_2^S$ is larger than $w_2^N$. With an increasing cost, the suppliers have to charge a higher wholesale price to prevent a loss.

Note that $K_1$ plays a role in making supplier 1’s wholesale price more or less sensitive to its unit manufacturing cost, because the opportunity cost is the function of the manufacturing cost. That $K_1 > 0$ means that with the availability of trade credit, supplier 1’s opportunity cost is greater than the retailer’s. As such, the loss that supplier 1 incurs is greater than the retailer’s profit and cannot be compensated by the interest that the retailer pays prompting supplier 1 to charge a higher price. Therefore, when $K_1 > 0$, supplier 1 solely extending trade credit raises product 1’s wholesale price closer to its unit manufacturing cost. However, when $K_1 < 0$, as the retailer’s opportunity cost is high, the interest that the retailer
pays is sufficient to offset the supplier’s loss, meaning that the supplier is willing to lower its charge, thus adversely affecting product 1’s wholesale price’s sensitivity to its cost.

By comparison, as supplier 2 requires the retailer to pay on delivery, it does not receive the interest paid by the retailer, and thus, it is not influenced by \( r_s \) and \( I_r \) directly. Interestingly, although the two products are complementary, the impact of \( K_1 \) on the sensitivity to cost of the two products’ wholesale prices is reversed because a positive \( K_1 \) increases the sensitivity of product 1’s wholesale price to its cost. This also raises the retail price, thereby reducing equilibrium demand for product 1. As product 2 is the complementary product, decreasing demand for product 1 also lowers demand for product 2. Hence, supplier 2 and the retailer have to drop their prices to stimulate market demand for product 2. Similarly to this analysis, when \( K_1 < 0 \), extending trade credit increases supplier 2’s wholesale price.

In contrast to Proposition 1, as supplier 1 extends trade credit, the sensitivity of the optimal retail price to product 2’s wholesale price is unchanged, whereas that to product 1’s wholesale price is altered. Comparing the retail prices in these two cases leads to the following Corollary 2.

**Corollary 2.** That a supplier unilaterally adopts a trade credit policy affects the retail prices of the products of both suppliers, as follows:

\[

p_1^S > p_1^N, \quad \text{if } K_1 > 0, \quad (25)
\]

\[

p_1^S < p_1^N, \quad \text{if } K_1 < 0. \quad (26)
\]

\[

p_2^S > p_2^N, \quad \text{if } K_1 (A_5 B_6 - A_6 B_5) > 0, \quad (27)
\]

\[

p_2^S < p_2^N, \quad \text{if } K_1 (A_5 B_6 - A_6 B_5) < 0. \quad (28)
\]

Corollary 2 suggests that the impact of trade credit on the retail price of product 1 in these scenarios is only related to the value of \( K_1 \). As the retail price is positively correlated with the wholesale prices of both products, when a supplier unilaterally extends trade credit, the impact of the changed sensitivity to product 1’s wholesale price is offset by the changed wholesale price resulting from the interest rate and opportunity cost. Thus, the impact of trade credit on product 1’s retail price only originates from the changed cost sensitivity of the wholesale prices. Further, because only supplier 1 adopts the trade credit policy, its effect on the wholesale price’s cost sensitivity is greater than that of supplier 2. Given that supplier 1’s wholesale price affects more on its own retail price than the complementary product’s wholesale price does, the effect of trade credit on product 1’s retail price is consistent with the impact on its own wholesale price from \( K_1 \). Therefore, when \( K_1 > 0 \), trade credit raises the retail price, but when \( K_1 < 0 \), the retail price falls. Nonetheless, product 2’s retail price is more affected by its own wholesale price and supplier 1’s trade credit policy influences supplier 2’s wholesale price to a less extent than its own.

**4.3 Both suppliers extending trade credit (Case D)**

Similarly, using a game-theoretical approach, the wholesale prices and retail prices of the two products in case D can be derived as given in Proposition 3.

**Proposition 3.** When both suppliers extend trade credit, the optimal wholesale prices \( w_1^D, w_2^D \)
and equilibrium retail prices \( p_1^D, p_2^D \) are respectively given as

\[
\begin{align*}
\hat{w}_1^D & = \frac{2(1+r_1)(1+r_2)(B_1B_5-B_2B_4)+ (1+r_2)c_1B_2B_5K_1-(1+r_1)c_2B_3B_5K_2}{2(1+r_1)(1+r_2)(1+r_1-I_r)(B_2B_5-B_3B_6)}, \\
\hat{w}_2^D & = \frac{2(1+r_1)(1+r_2)(B_2B_4-B_1B_6)+(1+r_2)c_2B_2B_6K_1-(1+r_1)c_1B_2B_6K_2}{2(1+r_1)(1+r_2)(1+r_2-I_r)(B_2B_5-B_3B_6)}, \\
\end{align*}
\]

(29)

(30)

\[
\begin{align*}
p_1^D & = A_1 + (1 + r_1 - I_r)A_2\hat{w}_1^D + (1 + r_2 - I_r)A_3\hat{w}_2^D, \\
p_2^D & = A_4 + (1 + r_2 - I_r)A_5\hat{w}_2^D + (1 + r_1 - I_r)A_6\hat{w}_1^D, \\
\end{align*}
\]

(31)

(32)

where \( A_1, A_2, \ldots, A_6, B_1, B_2, \ldots, B_6, K_1, K_2 \) are constants, as defined in Appendix A.

Similar to Proposition 2, the wholesale prices of both products are more affected by the relation between the individual interest rate that the retailer pays and the retailer’s opportunity cost. Owing to the complexity of this case, the effect of the trade credit policy is difficult to obtain. In particular, the manufacturing costs of both suppliers play a part in this case, with the extent dependent on the values of both \( K_1 \) and \( K_2 \). The numerical examples shown in Fig. 1 and Fig. 2 compare the optimal wholesale prices in the three cases, where the default values of the parameters involved are: \( \beta_{11} = 0.5, \beta_{22} = 0.4, \beta_1 = 0.45, \beta_2 = 0.35, \beta_{21} = 0.3, \beta_{12} = 0.25, a_1 = 100, a_2 = 80, c_1 = 30, c_2 = 25, r_1 = 0.04, r_2 = 0.04, I_1 = 0.13, I_2 = 0.08. \)

These two figures jointly reveal that when the trade credit policy is adopted, both suppliers’ equilibrium wholesale prices increase with the retailer’s opportunity cost. However, the wholesale prices under the trade credit policy do not always increase or decrease as compared to those achievable when no party extends trade credit. Specifically, for supplier 1, when \( I_r \) is small, extending trade credit reduces both products’ wholesale prices, and the two suppliers offering credit simultaneously drag the prices down further. This finding can be explained as that as the retailer’s profit falls with a low opportunity cost, the amount that the retailer can accept to pay the supplier also drops and the equilibrium wholesale price decreases. However, when \( I_r \) reaches a certain value, extending trade credit saves more opportunity cost and thus generates more profit for retailer 1, raising the equilibrium wholesale prices that the retailer pays the suppliers. Fig. 1 and Fig. 2 also illustrate that the impact of trade credit on product 2’s wholesale price reveals the same trend as that on product 1. As these two products are complementary, demands for both products should change in the same direction. Therefore, the price response also changes in the same way.
Proposition 3 shows that when both suppliers adopt a trade credit policy, the equilibrium retail price of each product is affected by the relation between the interest rate and the retailer’s opportunity cost, $K_1$ and $K_2$, with the solutions dependent upon different factors. Thus, it is difficult to compare the solutions in Case D with those in Case N or Case S directly.
However, as with Corollary 2, the impact on both products’ retail prices only originates from the wholesale prices’ changed cost sensitivity of the two products.

The numerical examples in Fig. 3 and Fig. 4 illustrate that both suppliers providing trade credit affects the wholesale prices and retail prices of both products. The default parameter values are the same as those given in Proposition 3.

Figs. 3 and 4 show that the impacts of trade credit on the two products’ retail prices with respect to $I_r$ are different. For product 1, the retail price with trade credit decreases with $I_r$, and when $I_r$ is sufficiently large, extending trade credit decreases product 1’s retail price. This may be attributed to the fact that a higher value of $I_r$ means that the retailer can save a larger amount of the opportunity cost when trade credit is provided, thus augmenting its total profit. As a result, the retailer is more motivated to decrease its price to stimulate demand. Meanwhile, when $I_r$ reaches a certain value, the retail price of product 1 in Case D is higher than it is in Case S because both suppliers extending trade credit to the retailer overstates the retailer’s emphasis to repay the loan from those upstream creditors, and the retailer thus has to raise the retail price to maintain its revenue.

![Fig. 3. Effect of $I_r$ on product 1’s retail prices in all three cases](image)
However, the case differs for product 2, whose retail price increases along with $I_r$ in Case S whilst decreasing with $I_r$ in Case D. Further, when $I_r$ is small, supplier 1 solely extending trade credit reduces product 2’s retail price. Yet, when the two suppliers adopt trade credit simultaneously, this raises product 2’s retail price, and when $I_r$ reaches a certain value, the impact on retail prices is reversed.

5. Comparisons and managerial implications

In this section, we compare the profits of the two duopolistic suppliers and monopolistic retailer in the aforementioned three scenarios to determine the circumstances under which trade credit is the most valuable to each firm. We also examine the impacts of selected parameters on the value of trade credit to identify managerial implications. The parameters in all three scenarios are assumed to be symmetric in order to simplify the case, avoiding possible problems caused by asymmetry. This follows the work by Tsay and Agrawal (2000) and Mishra and Raghunathan (2004), that is, $a_1 = a_2$, $\beta_1 = \beta_2$, $\beta_{11} = \beta_{22}$, $\beta_{12} = \beta_{21}$, $c_1 = c_2$, $l_1 = l_2$, $r_1 = r_2$, and $K_1 = K_2$. To facilitate comparisons, we set $\alpha = a + a_1 = a + a_2$, $\beta = \beta_1 + \beta_{11} = \beta_2 + \beta_{22}$, $\lambda = \beta_{12} = \beta_{21}$, $c = c_1 = c_2$, $l = l_1 = l_2$, and $r = r_1 = r_2$. In such a symmetric structure, $K_1 = K_2$, and we set this to $K$.

**Theorem 1.** Under different circumstances, the impact of the trade credit policy on supply chain parties’ profits is illustrated as follows:

$$\pi_i^D < \pi_i^S < \pi_i^N, \quad \text{if} \quad K > 0,$$
\[ \pi_i^N < \pi_i^S < \pi_i^D, \quad \text{if} \quad K < 0, \]

where \( i = 1, 2, \) and \( r. \)

Theorem 1 indicates that the profitability of extending trade credit is not definitely positive to supply chain parties. Instead, it is related to the value of \( K, \) which is the adjusted difference between the opportunity cost of the upstream supplier with the trade credit policy and that of the downstream retailer. And the profits of the three supply chain participants show the same trend in all three cases.

Firstly, with respect to supplier 1, when \( K \) is positive, extending trade credit to the retailer reduces supplier 1’s profit, and supplier 1 loses even more when it promotes a trade credit policy with supplier 2 simultaneously. However, when \( K \) is negative, extending trade credit can increase supplier 1’s profit, and the trade credit policy shows to be more worthwhile when the two suppliers adopt it simultaneously. Specifically, compared with when no party extends trade credit, when \( K \) is positive, supplier 1 extending trade credit raises the retail prices of both products, thus triggering a large decrease in demand for its own product. Meanwhile, as supplier 1’s opportunity cost also relates to the product quantity, the decreased demand raises the opportunity cost burden for supplier 1 markedly as well. Thus, although supplier 1 can increase the wholesale price it charges the retailer by adopting a trade credit policy, the trade credit effect is so weak that the corresponding profit rise owing to the hike in wholesale prices cannot cover the profit decline brought about by the reduced demand. Similarly, both suppliers adopting trade credit simultaneously causes a stronger effect on demand than on wholesale price resulting in a profit decrease. Conversely, when \( K \) is negative, supplier 1 allowing trade credit reduces (raise) the retail price of product 1 (2), causing a substantial expansion in demand for its own product. Thus, although the trade credit policy decreases the wholesale price, the profit increase due to the growing demand outweighs the loss from the lower wholesale price and the opportunity cost, and supplier 1’s profit increases ultimately.

Secondly, with respect to supplier 2, when \( K > 0, \) no party extending trade credit benefits supplier 2 the most; on the contrary, when \( K < 0, \) supplier 2 can maximize its profit by providing trade credit alongside supplier 1 simultaneously. Specifically, when \( K > 0, \) supplier 1 unilaterally extending trade credit cuts the wholesale price of product 2 according to Corollary 1. Meanwhile, supplier 1’s behaviour is to raise its retail price to a much greater extent and, as the two products are complementary, product 1’s highly increased retail price also decreases supplier 2’s demand, resulting in a lower profit for supplier 2. Similarly, when \( K < 0, \) supplier 1 providing credit to the retailer can enhance both the demand and the wholesale price of supplier 2, raising its profit. In the symmetric setting, the impact of both suppliers extending trade credit on supplier 2 is the same as that on supplier 1. With a positive \( K, \) the possible profit generated by the wholesale price increase is insufficient to cover the loss from the fall in demand. By contrast, when \( K \) is negative, the demand expansion contributes more to increasing profit than the lower wholesale price does to reducing the profit.

Finally, with respect to retailer’s attitude toward trade credit, the impact of the trade credit policy on the profits of the retailer shows the same trend as on the suppliers’ profits; in other words, the trade credit policy is advantageous only when \( K \) is negative. Further, although the retailer can accept the trade credit policy from upstream suppliers only passively,
when the two suppliers adopt the same policy, it always benefits the retailer the most. Specifically, when \( K \) is positive, although supplier 1 extending trade credit increases the retail price of product 1, the demand decreases significantly as price increases, which reduces the retailer’s unit profit of product 1. Also, the raised retail price decreases demand for product 2 due to their complementary nature. At the same time, although product 2’s unit profit increases as compared to Case N, as mentioned above, the demand decreases because of product 1’s increased retail price, which shrinks the retailer’s profit. On the contrary, the retailer’s profit rises when \( K \) is negative. In Case D, with a positive \( K \), both products’ unit profit and demand fall, causing the retailer’s profit to drop below that achievable in both Case N and Case S. The opposite holds when \( K \) is negative.

In summary, it can be derived that when \( K > 0 \), implementing a trade credit policy decreases the profits of both duopolistic suppliers as well as the monopolistic retailer; on the contrary, when \( K < 0 \), the two duopolistic suppliers extending trade credit simultaneously augment the profit of the entire supply chain. Furthermore, to maximize the profitability of every supply chain member, the two suppliers should always adopt the same trade credit strategy. Specifically, when \( K > 0 \), no party extending trade credit is more profitable, while when \( K < 0 \), the optimal strategy is for both suppliers to adopt the trade credit policy.

6. Numerical examples

To illustrate the impact of trade credit on the efficiency of the supply chain, we provide numerical examples in this section to (i) compare the profits of the two suppliers with that of the retailer in the two scenarios, and to (ii) present the optimal trade credit policy based on the different opportunity costs of the suppliers and the retailer.

6.1 Comparison between cases

In this subsection, to explain the results we have obtained and show the effect of retailer’s opportunity cost on supply chain members’ profit, we consider illustrative example as shown in Fig. 5, where the default parameters values are empirically set to \( \alpha = 180, \beta = 0.7, \lambda = 0.3, c = 30, I = 0.1 \) and \( r = 0.05 \). Fig. 5 presents the comparisons among the three cases. The setting of the parameter values consists with production practice and satisfies the assumptions made in Section 3, such as the condition that the realized demand is always nonnegative.

Fig. 5 depicts the profit changes of the two suppliers and the retailer with \( I_r \) in Case N, Case S and Case D, showing that when \( I_r \) reaches the value that reduces \( K \) below 0, the profits of all supply chain members in Case S are higher than those in Case N while lower than those in Case D and that they increase with the value of \( I_r \). These results suggest that when \( K \) is negative, both suppliers extending trade credit can raise the profits of all supply chain members while increasing the efficiency of the entire supply chain most, and the profit of parties will increase as \( I_r \) continues to increase. On the contrary, when \( K \) is positive, this behaviour will reduce the profit of every supply chain member more than that of only one supplier extending trade credit, and either behaviour will lower the efficiency of the supply chain.
With respect to suppliers, with a negative $K$, we find: 1) when no trade credit is provided, the profits of the suppliers are constant; 2) in the case of the two suppliers extending trade credit simultaneously both suppliers and the retailer are more profitable than that of only one supplier doing so; 3) at the same time, as the other party producing complementary products, supplier without providing trade credit only benefits from a quite small portion of the profits from trade credit policy. However, when $I_r$ reaches the value that makes $K$ positive, any supplier’s provision of trade credit will lead to a reduction in its own profits, which in turn will bring about a decrease in the efficiency of the supply chain. In other words, when one supplier adopts the trade credit policy, the other one behaving similarly would cause all members of the chain to suffer a loss. With respect to the retailer, when $I_r$ is above the threshold, either or all of suppliers extending trade credit can benefit not only themselves, but also the retailer, whereas a positive $K$ reduces the profits of all supply chain members and thus supply chain efficiency.

6.2 Trade credit strategy preference

In this subsection, we present the optimal trade credit strategy involving different opportunity costs for suppliers and the retailer, where the default parameters values are $\alpha = 180$, $\beta = 0.7$, $\lambda = 0.3$, $c = 30$, and $r = 0.05$. The result is shown in Fig. 6, reflecting that the suppliers’ and retailer’s opportunity costs influence the choice of trade credit strategy. In this figure, the “Case D zone” denotes the area in which the two suppliers extending trade credit simultaneously is the most profitable strategy, and the “Case N zone” denotes the area in which the supply chain is the most efficient when no party extends trade credit. This figure also illustrates that the case of only one supplier allowing trade credit is never the best choice for the supply chain. Further, with the suppliers’ opportunity cost fixed, the higher the retailer’s opportunity cost, the greater the profit when both suppliers extend trade credit. On the contrary, when the retailer’s opportunity cost is fixed, the effect of no party adopting the trade credit policy increases along with the suppliers’ opportunity cost.
7. Extensions

In this section, we first consider the opportunity cost of the retailer in the previously identified three scenarios and then, set the model parameters differently from those given in Section 5 in an effort to draw new conclusions regarding the performance of the suppliers and retailer. Further, we calculate the profits of both suppliers and the retailer when the product costs are no longer equal (i.e., $c_1 \neq c_2$).

To enable this comparative study, in Section 7.1, we simplify the parameters in the three scenarios by making them all symmetric, which consistent with the parameters in Section 5. We then explore the impact of the suppliers having asymmetric costs in Section 7.2. As reflected by the results that follow, our main conclusions are qualitatively unchanged.

7.1 Impact of trade credit policy with opportunity costs of both retailer and suppliers

In this subsection, considering opportunity costs of both the retailer and the two suppliers, we adopt a new profit function and compare all parties’ profits in the three cases, in order to determine find the optimal trade credit strategy.

Here, for the retailer, when no party extends trade credit, its profit is

$$\pi_r = (p_1 - w_1)q_1 + (p_2 - w_2)q_2 - I_r(w_1q_1 + w_2q_2). \quad (36)$$

When only one supplier extends trade credit, we assume that it is supplier 1. The profit of the retailer can be given as

$$\pi_r = [p_1 - (1 + r_1 - I_r)w_1]q_1 + (p_2 - w_2)q_2 - I_rw_2q_2. \quad (37)$$

When both suppliers adopt the trade credit policy, the retailer’s profit is

$$\pi_r = [p_1 - (1 + r_1 - I_r)w_1]q_1 + [p_2 - (1 + r_2 - I_r)w_2]q_2. \quad (38)$$
Theorem 2. Considering opportunity costs of all parties, the impact of the trade credit policy on supplier 1’s profit is given as follows:

\[ \pi_1^D < \pi_1^S < \pi_1^N, \text{ if } I_r \in (0, M_1) \cup (M_3, 1) \cap (0, M_2), \]

\[ \pi_1^N < \pi_1^S < \pi_1^D, \text{ if } I_r \in (M_1, M_3) \cap (M_2, 1). \]

where \( M_1, M_2, M_3 \) are constants, as defined in Appendix A.

Theorem 2 is basically the same as that presented earlier; however, when supplier 1 provides trade credit to the retailer, the supplier’s profit is only related to cost \( c \) and there are no additional conditions of \( I_r \) for this result. In other words, when the cost is within a certain range, it is always beneficial for the suppliers to provide trade credit when taking the opportunity cost of the retailer into consideration. In particular, if \( I_r \in (M_1, M_3) \cap (M_2, +\infty) \), extending trade credit to the retailer raises supplier 1’s profit. Supplier 1 will earn even more profit when supplier 2 provides trade credit simultaneously. When \( I_r \) is in the complementary area, the opposite is true.

When \( I_r \in (0, M_1) \cup (M_3, +\infty) \cap (0, M_2) \), supplier 1 extending trade credit will cause a loss. The rising retail prices of both products dramatically decrease the product’s demand. Although this lower demand can reduce the opportunity costs, the effect is too weak to compensate for the profit decline driven by the reduced demand, causing the profit to drop ultimately. Conversely, the effect of trade credit allows demand to surge to a high level, and the resulting consequent profit increase compensates for the loss from the lower wholesale price.

Theorem 3. Considering opportunity costs of all parties, the impact of the trade credit policy on supplier 2’s profit is given as follows:

\[ \pi_2^D < \pi_2^S < \pi_2^N, \text{ if } I_r \in (M_1, M_4) \cap (0, M_5), \]

\[ \pi_2^N < \pi_2^S < \pi_2^D, \text{ if } I_r \in (0, M_1) \cup (M_4, 1) \cap (M_5, 1), \]

where \( M_1, M_4, M_5 \) are constants as specified in Appendix A.

Theorem 3 guarantees the same result for profit as Theorem 2. Just as indicated by Theorem 1, when \( I_r \in (M_1, M_4) \cap (0, M_5) \), no party extending trade credit benefits supplier 2. However, when \( I_r \in (0, M_1) \cup (M_4, +\infty) \cap (M_5, +\infty) \), supplier 2’s profit decreases by adopting the trade credit policy alongside supplier 1 simultaneously.

Specifically, Theorem 3 shows that supplier 2’s profit decreases as supplier 1 provides trade credit. As the two products are complementary, supplier 1 providing trade credit reduces demand for the complementary products, resulting in a lower profit for supplier 2. Yet, compared with Case D, when \( I_r \in (M_1, M_4) \cap (0, M_5) \), if supplier 2 also adopts the trade credit policy, both products’ retail prices rise and thus, its demand shrinks substantially, thereby decreasing the profit to a larger extent. On the contrary, when \( I_r \) is in the opposite area, both product 1 and product 2 have lower prices and the decrease in both products’ retail prices triggers product 2’s demand, which help raise supplier 2’s profit.

Theorem 4. Considering opportunity costs of all parties, the impact of the trade credit policy on retailer’s profit is as follows:
\[ \pi_r^D < \pi_r^S < \pi_r^N, \text{ if } I_r \in (0, M_6) \cup (M_1, 1), \]
\[ \pi_r^N < \pi_r^S < \pi_r^D, \text{ if } I_r \in (M_7, M_1), \]

where \( M_1, M_6, M_7 \) are constants, as defined in Appendix A.

Theorem 4 indicates that the impact of the trade credit policy on the profits of the retailer shows the same trend as that for the suppliers’ profits. When \( I_r \in (M_7, M_1) \), the retailers receiving trade credit from the suppliers will increase its profit, and this rise becomes even larger when both suppliers offer trade credit.

When \( I_r \in (0, M_6) \cup (M_1, +\infty) \), although supplier 1 providing trade credit increases the retail price of product 1 (i.e., the retailer’s unit profit will rise), demand for product 1 falls simultaneously. At the same time, as suppliers 1 and 2 provide complementary products, the increase in the price of product 1 leads to the decline in the price of product 2, which reduces the retailer’s revenue further.

### 7.2 Impact of trade credit policy on profits of suppliers and retailer with asymmetric costs

In this subsection, we extend the parameters setting to a more general, and demonstrate the credit policy impact of all parties having asymmetric costs (i.e., \( c_1 \neq c_2 \)). Moreover, we also explore the specific impact of the retailer by assigning specific values to the parameters and drawing a figure.

**Theorem 5.** Under asymmetric costs setting, the impact of the trade credit policy on supplier 1’s profit is illustrated as follows:

\[
\pi_1^D < \pi_1^S < \pi_1^N, \quad \text{if } c_1 \in (G_1, G_2) \cap (G_3, G_4),
\]
\[
\pi_1^D > \pi_1^S > \pi_1^N, \quad \text{if } c_1 \in (0, G_1) \cup (G_2, G_4) \cap (0, G_3),
\]

where \( M_1, M_6, M_7 \) are constants (defined in Appendix A).

Theorem 5 shows that in an asymmetric cost setting, the participant’s profit is only related to the relationship between cost 1 and cost 2. Specifically, comparing Case N with Case S, we find that the difference in their profit functions is a quadratic equation, which is related to \( c_1 \). If \( c_1 \in (G_1, G_2) \), \( \pi_1^S < \pi_1^N \) and if \( c_1 \in (0, G_1) \cup (G_2, G_4) \), \( \pi_1^S > \pi_1^N \). However, comparing the results on Case S with Case D shows that the difference between the two profit functions is a linear function of \( c_1 \). Further, if \( c_1 > G_3, \pi_1^D < \pi_1^N \) and if \( 0 < c_1 < G_3, \pi_1^D > \pi_1^N \). Hence, compared with the extension described in Section 7.1, the establishment of Theorem 5 requires a significantly less number of conditions (e.g., without involving the opportunity cost and interest rate).

**Theorem 6.** Under asymmetric costs setting, the impact of the trade credit policy on supplier 2’s profit is given as follows:

\[
\pi_2^D < \pi_2^S < \pi_2^N, \quad \text{if } I_r \in \left(0, \frac{(1+r)(1+i)}{2}\right),
\]
\[
\pi_2^D > \pi_2^S > \pi_2^N, \quad \text{if } I_r \in \left(\frac{(1+r)(1+i)}{2}, 1\right).
\]

Theorem 6 shows the same conclusion as that drawn in Section 7.1. This limits the range
of values for the retailer’s opportunity cost. Indeed, the condition for the establishment of Theorem 6 is the same as that for conducting the comparison of Case N with Case S. When \( I_r \in \left(0, \frac{(1+r)(1+l)}{2}\right)\), trade credit being provided by supplier 1 decreases the demand of supplier 2 and thus reduces supplier 2’s profit. This effect is more serious in Case D where we have \( \pi_2^S < \pi_2^N \). When \( I_r \in \left(\frac{(1+r)(1+l)}{2}, 1\right)\), the provision of trade credit by supplier 1 has a positive impact on supplier 2 and this effect is more pronounced in Case D, which means \( \pi_2^S > \pi_2^N \). However, at the same time, the comparison between Case S and Case D is done without involving any conditions other than the cost. Note that the cost function similar to Theorem 5 is a linear function about itself and the difference is a quadratic function.

By comparing the retailer’s profits under three scenarios, we found that the core conclusions have not changed. The impact of the trade credit policy on the profits of the retailer shows the same trend as that on supplier 1’s profits. Indeed, all the comparison results were established without a constraint, and like Theorem 5, there were few similar restrictions. We provide a numerical example to describe the comparison of retailer’s profits more intuitively, where the default parameter values of parameters are \( a = 180, \beta = 0.7, \lambda = 0.3, l = 0.1, r = 0.05, I_r = 0.1 \).

Fig. 7 depicts the profit changes of the retailer with \( c_1 \) in three cases. When \( c_1 \) is small, supplier 1 increases the price of product 1 to share the profit from the provision of trade credit. This leads to a decrease in demand of its product as well as a reduction in demand for product 2, leading to a lower profit margin for the retailer. This way of raising prices causes a further decline in the retailer’s profits in Case D, with \( \pi_r^D < \pi_r^S < \pi_r^N \). Therefore, as \( c_1 \) increases, the profit of retailer in case D exhibits a faster downward trend than that in case N. However, although the cost increase will reduce the demand, the increased retail price can compensate for this part of the loss for the retailer’s profit in Case S. As \( c_1 \) rises further, the result is the
opposite, namely $\pi_r^N < \pi_r^S < \pi_r^D$.

8. Conclusion

We have analyzed the pricing decisions in a supply chain that consists of two duopolistic suppliers who manufacture complementary products and a common monopolistic retailer under different scenarios in which both suppliers have the right to decide whether to extend trade credit to the retailer. A number of results have been established: First, three Bertrand models having been formulated respectively regarding the cases where: (i) no supplier adopts the trade credit policy, (ii) only one supplier extends trade credit, and (iii) both suppliers extend trade credit. Second, the optimal equilibrium wholesale prices and retail prices of the two complementary products in each of these scenarios have been identified and their differences compared. Third, given the assumption that these two complementary products are symmetric, the three firms’ equilibrium profits in various situations have also been compared with the optimal strategy under such circumstances derived.

Based on our analysis, several interesting interpretations and managerial implications are obtained. First, it has revealed that the equilibrium solutions with the trade credit policy are affected by both the difference between the interest rate that the retailer pays the creditor and the retailer’s opportunity cost, as well as the adjusted difference in the opportunity cost between the creditor and retailer. Furthermore, product cost has been verified to affect the pricing strategies of the parties in such a supply chain with trade credit in addition to those differences of both interest rate and opportunity cost. Moreover, with respect to whether suppliers should extend trade credit, we find that the difference in the opportunity cost has a consistent effect on whether parties in the supply chain adopt trade credit. In other words, when the value of $K$ is negative, both suppliers and retailers can benefit from the trade credit policy, and their profits increase with the number of suppliers that provide trade credit; Otherwise, forego trade credit is their best strategy. More concretely, in the symmetric case, this investigation has concluded that when the adjusted difference in the opportunity cost between the upstream and downstream actors is positive, the trade credit policy reduces the profits of all participants, whereas all members of the supply chain will benefit from the trade credit policy when the opportunity cost is negative. Furthermore, by comparing scenarios supported with numerical examples, this research has verified that to maximize the profit of all members and the efficiency of the entire supply chain, the two suppliers should always adopt the same trade credit strategy.

Our results are based on a series of assumptions, which deserve further examination. First, we have only analyzed supplier-controlled markets while establishing a Bertrand model. Dealing with the cases of a retailer-controlled market and two suppliers moving sequentially may generate different results from our study. Second, we have assumed that the opportunity cost holds a linear relationship with a firm’s profit. Further research could extend this setting to include a non-linear function of opportunity cost. Moreover, currently, demand has also been assumed to be linear without considering other factors that may influence it. This forms another aspect that could be further discussed in future research. Finally, when comparing the profits of the three parties in the various scenarios investigated, the two suppliers have been assumed to be symmetric. Yet, asymmetric situations may be more realistic in real-world problems, albeit addressing such situations may require much more challenging extensions.
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Appendix A. Notations

\[ A_1 = \frac{(b_{12} + b_{21})(a_2 + a) - 2(b_2 + b_{22})(a_1 + a)}{(b_{12} + b_{21})^2 - 4(b_1 + b_{11})(b_2 + b_{22})} \]
\[ A_2 = \frac{(b_{12} + b_{21})b_{12} - 2(b_1 + 2b_{22})(b_1 + b_{11})}{(b_{12} + b_{21})^2 - 4(b_1 + b_{11})(b_2 + b_{22})} \]
\[ A_3 = \frac{(b_{12} + b_{21})^2 - 4(b_1 + b_{11})(b_2 + b_{22})}{(b_{12} - b_{21})(b_2 + b_{22})} \]
\[ A_4 = \frac{(b_{12} + b_{21})(a_1 + a) - 2(b_1 + b_{11})(a_2 + a)}{(b_{12} + b_{21})^2 - 4(b_1 + b_{11})(b_2 + b_{22})} \]
\[ A_5 = \frac{(b_{12} + b_{21})b_{21} - 2(b_2 + b_{22})(b_1 + b_{11})}{(b_{12} + b_{21})^2 - 4(b_1 + b_{11})(b_2 + b_{22})} \]
\[ A_6 = \frac{(b_{12} + b_{21})^2 - 4(b_1 + b_{11})(b_2 + b_{22})}{(b_{12} - b_{21})(b_1 + b_{11})} \]
\[ B_1 = a_1 + a - (b_1 + b_{11})A_1 - b_{12}A_4 + (b_1 + b_{11})c_1A_2 + b_{12}c_1A_6 \]
\[ B_2 = 2(b_1 + b_{11})A_2 + 2b_{12}A_6 \]
\[ B_3 = (b_1 + b_{11})A_3 + b_{12}A_5 \]
\[ B_4 = a_2 + a - (b_2 + b_{22})A_4 - b_{21}A_1 + (b_2 + b_{22})c_2A_5 + b_{21}c_2A_3 \]
\[ B_5 = 2(b_2 + b_{22})A_5 + 2b_{21}A_3 \]
\[ B_6 = (b_2 + b_{22})A_6 + b_{21}A_2 \]
\[ K_1 = I_1(1 + r_1 - l_r) - l_r \]
\[ K_2 = I_2(1 + r_2 - l_r) - l_r \]
\[ R = (1 + r - l_r) \]
\[ K = I(1 + r - l_r) - l_r \]
\[ M_1 = \frac{I(1 + r)}{2 + I + r} \]
\[ M_2 = \frac{(1 + r)(2I - 1)}{1 + 2I + r} \]
\[ M_3 = \frac{(1 + r)(8\beta^2 - 4\lambda^2 - 3b_1 + 3b_2\lambda)}{b_1\lambda(1 + r) + b_2\lambda(1 - r) + 8\beta^2 - 4\lambda^2} \]
\[ M_4 = \frac{2\lambda^2(1 + r) + (1 + r)\beta\lambda - 4(1 + r)\beta^2}{(1 + r)(4\beta^2 + 2\beta_1 + 2\lambda_1 - 2\lambda^2)} \]
\[ M_5 = \frac{(1 + r)(1 - l_r)b_1\lambda(\lambda^2 - 2\beta^2) + \beta^2\lambda^2(4 + 2I + 4r - 3I_r) - r(4\beta^4 - \lambda^4) + H_1}{(1 + r)(1 + r + 2I)(\lambda^4 + 4\beta^4) + (1 - I)(1 + r)\beta\lambda(2\lambda^2 - 4\beta^2) - H_2} \]
\[ M_6 = \frac{(1 - r)\beta(4\beta^2 - 3\lambda^2) + 2\lambda^3}{(1 - r)\beta(4\beta^2 - 3\lambda^2) - 2\lambda^3(1 + r)} \]
\[ M_7 = \frac{(1 + r)(2 + 2I)\lambda^3 + (1 + r)(1 + r)\beta(4\beta^2 - 3\lambda^2)}{2(1 + r)\lambda^3 + (1 - r)\beta(4\beta^2 - 3\lambda^2)} \]
\[ H_1 = \sqrt{(2 + r)^2(2 + \lambda^2)(2\beta - \lambda^2)^2(3\lambda^2 \beta^2 + 21\lambda^3 - 4\lambda\beta^3 - 4\beta^2 - \lambda^4)} \]
\[ H_2 = (4 + 4I + 4r^2 + 8r + 6Ir)\beta^2\lambda^2 \]
\[ G_1 = \frac{E_1 - E_3}{\sqrt[2i(1 + r)R]{E_2}} \]
\[ G_2 = \frac{E_1 + E_3 \sqrt{\frac{l}{2(1+r)R}}}{E_2} \]
\[ G_3 = \frac{E_4 + 4(1-l)(1+r)a\beta}{4R(2\beta^2 - \lambda^2)} \]
\[ G_4 = \frac{2a\beta^2 + 2l\lambda^2 + \beta^2 \lambda c_2 + 4l\beta^2 - 2l\beta\lambda - a\beta\lambda}{\beta(2\beta^2 - \lambda^2)} \]

\[ E_1 = (1 + r)l(\beta\lambda c_2 + 2l\lambda - 2a\beta + a\lambda + 4\beta) \]
\[ E_2 = l(2l_r - I - 2r - lr - 2)(2\beta^2 - \lambda^2) \]
\[ E_3 = (R + l + lr - l_r)(\beta\lambda c_2 + a\lambda - 2a\beta) + 2\lambda(1 + r)(I - 1) - 4l(\lambda l_r + 2\beta R) \]
\[ E_4 = \beta\lambda c_2(2l_r + I - 3 + 3r + lr) + 2\lambda(1 + r)(al - 1 - l - a) \]

**Appendix B. Proofs**

**Proof of Proposition 1.**

From Eqs. (5), (6), and (9), the first-order partial derivatives of \( \pi_r(p_1, p_2) \) to \( p_1 \) and \( p_2 \) are

\[ \frac{\partial \pi_r(p_1, p_2)}{\partial p_1} = a_1 + a - 2(\beta_1 + \beta_{11})p_1 - (\beta_{12} + \beta_{21})p_2 + (\beta_1 + \beta_{11})w_1 + \beta_{21}w_2, \quad (b1) \]
\[ \frac{\partial \pi_r(p_1, p_2)}{\partial p_2} = a_2 + a - 2(\beta_2 + \beta_{22})p_2 - (\beta_{12} + \beta_{21})p_1 + (\beta_2 + \beta_{22})w_2 + \beta_{12}w_1. \quad (b2) \]

Then, we compute the second-order derivatives of Eqs. (b1) and (b2) to obtain the Hessian matrix:

\[ \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_1^2} = -2(\beta_1 + \beta_{11}), \quad \frac{\partial^2 \pi_r(p_1, p_2)}{\partial p_2^2} = -2(\beta_2 + \beta_{22}). \quad (b3) \]

Thus, from Eq. (b3), we conclude that the determinant of the Hessian matrix is positive. Therefore, \( \pi_r(p_1, p_2) \) is jointly concave in \( p_1 \) and \( p_2 \). Setting Eqs. (b1) and (b2) equal to zero and solving them simultaneously, we obtain the optimal prices of the two products in Case N:

\[ p_1^N(w_1, w_2) = A_1 + A_2 w_1 + A_3 w_2, \quad (b4) \]
\[ p_2^N(w_1, w_2) = A_4 + A_5 w_2 + A_6 w_1, \quad (b5) \]

where \( A_1, A_2, ..., A_6 \) are constants.

From Eqs. (11) and (12), the first-order partial derivatives of \( \pi_1 \) to \( w_1 \) and the first-order partial derivatives of \( \pi_2 \) to \( w_2 \) are

\[ \frac{\partial \pi_1}{\partial w_1} = (B_1 - B_2 w_1 - B_3 w_2)(1 + l_1). \quad (b6) \]
\[ \frac{\partial \pi_2}{\partial w_2} = (B_4 - B_5 w_2 - B_6 w_1)(1 + l_2). \quad (b7) \]

Then, to check the optimality, the second-order partial derivatives are respectively

\[ \frac{\partial^2 \pi_1(w_1, w_2)}{\partial^2 w_1} = -B_2(1 + l_1), \quad (b8) \]
\[ \frac{\partial^2 \pi_2(w_1, w_2)}{\partial^2 w_2} = -B_5(1 + l_2). \quad (b9) \]
From Eqs. (b8) and (b9), we know that \( \pi_1 \) and \( \pi_2 \) are concave in \( w_1 \) and \( w_2 \), respectively. Thus, by setting Eqs. (b6) and (b7) equal to zero and solving them simultaneously, Proposition 1 is proven.

**Proof of Proposition 2.** By substituting Eqs. (14) and (15) into Eqs. (b4) and (b5), Proposition 2 is obtained.

**Proof of Proposition 3.** The proof is similar to that of Proposition 1.

**Proof of Corollary 1.**

Comparing the wholesale prices of the two products in the three scenarios, we obtain the following relationships:

\[
\begin{align*}
\bar{w}_1^S &= \frac{w_1^N}{1+r_1-I_1} + \frac{c_1 B_2 B_5 K_1}{2(1+r_1)(1+r_2-I_2)(B_2 B_5-B_3 B_6)} \\
\bar{w}_2^S &= w_2^N - \frac{c_1 B_2 B_6 K_1}{2(1+r_1)(1+r_1-I_2)(B_2 B_5-B_3 B_6)}
\end{align*}
\] (b10)

Under the assumption that the self-price sensitivities are greater than the cross-price sensitivities and that all the parameters are positive, we can verify that \( B_1, B_2,...,B_6 \) are all positive constants and \( B_2 - B_3 > 0, B_5 - B_6 > 0 \). Thus, from Eqs. (b10) and (b11), Corollary 1 is obtained.

**Proof of Corollary 2.**

Comparing the retail prices of the two products in the three cases, the following relationships are obtained:

\[
\begin{align*}
\bar{p}_1^S &= p_1^N + \frac{c_1 B_2 (A_2 B_5-A_3 B_6) K_1}{2(1+r_1)(B_2 B_5-B_3 B_6)} \\
\bar{p}_2^S &= p_2^N - \frac{c_1 B_2 (A_3 B_5-A_4 B_6) K_1}{2(1+r_1)(B_2 B_5-B_3 B_6)}
\end{align*}
\] (b12)

Under the assumption that the self-price sensitivities are greater than the cross-price sensitivities, \( B_1, B_2,...,B_6 \) are all proven to be positive constants and \( B_5 - B_6 > 0 \). Thus, from Eqs. (b12) and (b13), Corollary 2 is obtained.

**Proof of Theorem 1.**

Given symmetric structure of the case with the following setting: \( \alpha = a + a_1 = a + a_2 \), \( \beta = \beta_1 + \beta_{11} = \beta_2 + \beta_{22} \), \( \lambda = \lambda_1 = \lambda_2 \). \( c = c_1 = c_2 \), \( I = I_1 = I_2 \) and \( r = r_1 = r_2 \), we compute the difference in supplier 1’s profit between Case N and Case S such that

\[
\begin{align*}
\pi_1^S - \pi_1^N &= \frac{1}{2} \beta \left[ (4K(1+r)B^2 - K(K+5)r+5)\beta^2 \lambda^2 + K(1+r-I_r)\lambda^4 c^2 + [4aK(1+r)\beta^2 \lambda - 2aD(1+r)\beta^2 \lambda^2] c - 4a^2 K(1+r)\beta^2 + 4a^2 K(1+r)\beta \lambda - \alpha^2 K(1+r)\lambda^2 \right].
\end{align*}
\] (b14)

which is a one-variable quadratic polynomial about \( c \). Under the assumption that the self-price sensitivity is larger than the cross-price sensitivity, we know that \( 4K(1+r)\beta^4 + K(4K + 5r + 5)\beta^2 \lambda^2 - K(1+r-I_r)\lambda^4 > 0 \) when \( K > 0 \) and \( 4K(1+r)\beta^4 + K(4K + 5r + 5)\beta^2 \lambda^2 - K(1+r-I_r)\lambda^4 < 0 \) when \( K < 0 \).
Then, we calculate the $\Delta$ of the divisor of Eq. (b14):

$$
\Delta = 4\alpha^2 K^2 (2\beta - \lambda)^2 (2\beta^2 - \lambda^2)^2 (1 + r)(K + 1 + r),
$$

(b15)

which is positive. Thus, both $\Delta$ and the coefficient of the quadratic term of the divisor of Eq. (b14) are positive. With the restriction that $c > 0$ and the dividend of Eq. (b14) is positive, the following holds:

When $K > 0$,

$$
\pi_S^I > \pi_N^I, \text{ if } c > \frac{2a(1+r)(2\beta^2 - \lambda^2) + 2a(2\beta^2 - \lambda^2)(2\beta - \lambda)\sqrt{(1+r)(K+1+r)}}{2(K+1+r)(2\beta^2 - \lambda^2)^2 + (1+r)\beta^2 \lambda^2},
$$

(b16)

When $K < 0$,

$$
\pi_S^I > \pi_N^I, \text{ if } 0 < c < \frac{2a(1+r)(2\beta^2 - \lambda^2) + 2a(2\beta^2 - \lambda^2)(2\beta - \lambda)\sqrt{(1+r)(K+1+r)}}{2(K+1+r)(2\beta^2 - \lambda^2)^2 + (1+r)\beta^2 \lambda^2}.
$$

(b17)

Similarly, we compute supplier 1’s profit changes in Case D and compare that with Case N and Case S. As $\beta > 2\lambda$ and all the parameters are positive, we have:

When $K > 0$,

$$
\pi_D^I > \pi_N^I, \text{ if } c > \frac{a(1+r)}{\beta + \lambda} \sqrt{\frac{1+r}{K+1+r}},
$$

(b18)

and

$$
\pi_S^I > \pi_D^I, \text{ if } 0 < c < \frac{2a(1+r)(2\beta - \lambda)}{(K+1+r)(4\beta^2 + \beta\lambda - 2\lambda^2) + (1+r)\beta^2 \lambda}.
$$

(b19)

When $K < 0$,

$$
\pi_D^I > \pi_N^I, \text{ if } 0 < c < \frac{a(1+r)}{\beta + \lambda} \sqrt{\frac{1+r}{K+1+r}},
$$

(b20)

and

$$
\pi_S^I > \pi_D^I, \text{ if } c > \frac{2a(1+r)(2\beta - \lambda)}{(K+1+r)(4\beta^2 + \beta\lambda - 2\lambda^2) + (1+r)\beta^2 \lambda}.
$$

(b21)

To ensure the model is realistic and meaningful, we need to guarantee that the demand level in each scenario is positive. Then, we obtain the following limitations:

When $K > 0$,

$$
q > 0 \text{ if } c < \frac{a(1+r)}{(\beta + \lambda)(D+1+r)},
$$

(b22)

When $K < 0$,

$$
q > 0 \text{ if } c < \frac{a(1+r)(2\beta - \lambda)}{(1+r)(4\beta^2 + \beta\lambda - 2\lambda^2) + K\beta \lambda}.
$$

(b23)

Using Eqs. (b16)–(b23), Theorem 1 is proven.

**Proof of Theorem 2.**

As we consider the symmetric structure of the case and set $\alpha = a + a_1 = a + a_2$, $\beta = \beta_1 + \beta_{11} = \beta_2 + \beta_{22}$, $\lambda = \beta_{12} = \beta_{21}$, $I = I_1 = I_2$, $r = r_1 = r_2$, and $c = c_1 = c_2$, we compute the difference in supplier 1’s profit between Case S and Case D:

$$
\pi_S^I - \pi_D^I = \frac{\lambda \beta^2 (1+l)[2+3+l(r-I)] + 2\lambda \beta[2\beta^2 - \lambda^2][2\beta - \lambda]r(1+l)}{4(\beta^2 + \lambda^2)^2(1+r)(1+r-Ir)} \cdot c^2 - \frac{1}{4(\beta^2 + \lambda^2)^2(1+r)(1+r-Ir)} \lambda \beta (1+l)[\beta[r(I-I_r) + I - I_r(2+l)][4(1 + r)(I-1)\alpha \beta +}
$$
When $\pi_1$ and $\pi_1'$ are compared in Case N and Case D, we compute supplier 1's profit changes in Case N and compare that with Case D.

Similarly, we compute supplier 1's profit changes in Case N and compare that with Case D and Case S. As $\beta > 2\lambda$ and all the parameters are positive, we have:

$$M_1 = \frac{(1+r)}{2+I+I+I} \quad \text{and} \quad M_3 = \frac{(1+r)}{\lambda(I+I+I)+\lambda(I+I+I)+\lambda^2}$$

Then, we calculate the $\Delta$ of Eq. (b24):

$$\Delta = \frac{\beta^2}{(2\beta+\lambda)^2(1+I+I+I+I)}$$

which is positive. With the restriction that $c > 0$ and $0 < I_r < 1$, we find that:

When $I_r \in (0, M_2)$,

$$\pi_1^N > \pi_1^P, \text{ if } c > \frac{2\alpha\beta(1+I+I+I+I)}{\alpha(1+I+I+I+I)+\lambda(1+I+I+I+I)}$$

and

$$\pi_1^D > \pi_1^N, \text{ if } 0 < c < \frac{\lambda(1+I+I+I+I)}{(2\beta^2+\lambda^2)(1+I+I+I+I+I+I+I)}$$

When $I_r \in (M_2, 1)$,

$$\pi_1^N > \pi_1^P, \text{ if } 0 < c < \frac{2\alpha\beta(1+I+I+I+I)}{\alpha(1+I+I+I+I)+\lambda(1+I+I+I+I+I+I+I)}$$

and

$$\pi_1^D > \pi_1^N, \text{ if } c > \frac{\lambda(1+I+I+I+I)}{(2\beta^2+\lambda^2)(1+I+I+I+I+I+I+I)}$$

To ensure the model is realistic and meaningful, we need to guarantee that the demand level in each scenario is positive. Then, we obtain the following limitations:

When $I_r \in (0, M_1) \cup (M_3, 1) \cap (0, M_2)$,

$$q > 0 \text{ if } c < \frac{\alpha\beta(1+I+I+I+I)}{(1+I+I+I+I)}$$

When $I_r \in (M_1, M_3) \cup (M_2, 1)$,

$$q > 0 \text{ if } c < \frac{\alpha\beta(1+I+I+I+I)}{2R(1+I+I+I+I)}$$

Using Eqs. (b23)–(b34), Theorem 4 is proven.

Proof of Theorem 3. The proof is similar to that of Theorem 2.

Proof of Theorem 4. The proof is similar to that of Theorem 2.
Proof of Theorem 5.
As we consider the symmetric structure of the case and set $\alpha = a + a_1 = a + a_2$, $\beta = \beta_1 + \beta_{11} = \beta_2 + \beta_{22}$, $\lambda = \beta_{12} = \beta_{21}$, $I = l_1 = l_2$, $r = r_1 = r_2$, but set $c_1 \neq c_2$, we compute the difference in supplier 1’s profit between Case S and Case D:

$$\pi_1^S - \pi_1^D = \frac{\lambda \beta_1^2 [2(1+\lambda)(1-r) + \beta_1^2 - 2(1+\lambda)(1+r)]}{\beta(2\beta - \lambda)^2 (2\beta + \lambda)^2} \{ -4(2\beta^2 - \lambda^2)(1 + r - I_r)c_1 + \lambda[(1 + r)(I - 3) + 2l_r]c_2 + 4\alpha \beta_1 (1 - I)(1 + r) - 2\lambda(1 + r)(1 + \alpha)(1 - I) \}$$  \quad (b35)

Which is a one-variable linear polynomial about $c_1$. With the restriction that $c > 0$, we find that:

$$\pi_1^S > \pi_1^D, \quad \text{if } c_1 > \frac{c_2[2l_r + (1+r)(I-3)]\beta + 2(1+r)[l(a - 1) + (1+a)]}{(1+r-I_r)(\beta^2 - 4\lambda^2)}$$  \quad (b36)

and

$$\pi_1^S < \pi_1^D, \quad \text{if } 0 < c_1 < \frac{c_2[2l_r + (1+r)(I-3)]\beta + 2(1+r)[l(a - 1) + (1+a)]}{(1+r-I_r)(\beta^2 - 4\lambda^2)}$$  \quad (b37)

Similarly, we compute supplier 1’s profit changes in Case N and compare that with Case D and Case S. As $\beta > 2\lambda$ and all the parameters are positive, we have:

$$\pi_1^N > \pi_1^S, \quad \text{if } c \in \left( \frac{E_1 - E_3 \sqrt{I - 2(1+r)R}}{E_2}, \frac{E_1 + E_3 \sqrt{I - 2(1+r)R}}{E_2} \right)$$  \quad (b38)

$$\pi_1^D > \pi_1^N, \quad \text{if } c_1 > \frac{c_2 \beta [2l_r + (1+r)(I-3)]\beta + 2(1+r)[l(a - 1) + (1+a)]}{2\lambda^2 \beta^2 (1+r)(1+r-I_r)}$$  \quad (b39)

and

$$\pi_1^S > \pi_1^N, \quad \text{if } c \in \left( 0, \frac{E_1 - E_3 \sqrt{I - 2(1+r)R}}{E_2} \right) \cup \left( \frac{E_1 + E_3 \sqrt{I - 2(1+r)R}}{E_2}, +\infty \right)$$  \quad (b40)

$$\pi_1^N > \pi_1^D, \quad \text{if } c_1 < \frac{c_2 \beta [2l_r + (1+r)(I-3)]\beta + 2(1+r)[l(a - 1) + (1+a)]}{2\lambda^2 \beta^2 (1+r)(1+r-I_r)}$$  \quad (b41)

To ensure the model is realistic and meaningful, we need to guarantee that the demand level in each scenario is positive. Then, we obtain the following limitations:

$$q > 0 \quad \text{if} \quad c < \frac{2\alpha \beta^2 + 2l_k^2 + 2\beta^2 x_2 + 4\beta^2}{\beta(2\beta^2 - \lambda^2)}$$  \quad (b42)

Using Eqs. (b35)–(b42), Theorem 5 is proven.

Proof of Theorem 6. The proof is similar to that of Theorem 5.

References


