# The transition from three-dimensional to two-dimensional foam structures

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# The transition from three-dimensional to two-dimensional foam structures

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**Abstract.** A two-dimensional foam consists of a monolayer of bubbles. It can be created by squeezing the more familiar three-dimensional foam between two parallel glass plates. We describe and explain the minimum plate separation $H$ which must be reached to fully effect the transition from three- to two-dimensional foam. We find that $H/V^{1/3}$ is close to one, where $V$ is the average bubble volume, and increases slightly when the side-walls of the container are taken into account.

## 1 Introduction

Foams are found in a variety of industrial and domestic situations, where they are used for many purposes, including cleaning and ore separation [1]. In order to validate theory and simulation tools, it is common to work with two-dimensional foams [2]. In this case, Plateau's rules of foam geometry are simplified and an experimental realization is readily available: a single layer of bubbles, such as can be made on the surface of a liquid or between two parallel surfaces. It is therefore important to know for which ranges of parameters, including the bubble volume $V$ and the surface separation $H$, a 2D foam can be sustained.

Cox *et al.* [3] described an instability in which a single layer of soap bubbles confined between two parallel glass plates undergoes a spontaneous transition to a two-layer structure. This is triggered for a bubble with $N$ sides when the separation of the plates exceeds the critical value $H_{max} = \sqrt[3]{6V\pi/(6-N)}$. Thus three- and then four-sided bubbles undergo the transition first (as $H/V^{1/3}$ is increased above values of around 1.8 and 2 respectively), while bubbles with 6 or more sides are indefinitely stable. This 2D to 3D transition is a surface-tension driven instability, triggered by a Rayleigh-Plateau mechanism, which can be predicted by noting that soap films minimize their surface area [4]. The predictions agree with experiments [3] and thus provide a means to predict the maximum plate separation to keep an experiment two-dimensional.

Here, we study the reverse transition: when does a foam consisting of two layers of bubbles revert to a single layer as the separation is decreased? That is, how much must a 3D foam be squeezed to ensure that it will become 2D? Rather than being of Rayleigh-Plateau type, this 3D to 2D instability is triggered by a tilting of the soap film that separates two bubbles horizontally (i.e. parallel to the glass plates), as discussed previously for two isolated bubbles [5,6].

We first describe experiments in a Hele-Shaw cell that illustrate the effect. An idealized arrangement of bubbles is then simulated to determine the critical separations. Finally, we characterize the instability theoretically, as a function of the number of sides of the horizontal inter-bubble film, to provide a guide to the critical separation.

## 2 Experiments

Experiments were carried out using a Hele-Shaw cell with varying plate separation. The wedge-shaped walls of the cell were machined from Carp Brand Tufnol ® to give a plate separation varying from $H = 4.4$mm at the entrance of the cell to $H = 1.5$mm a distance 500mm into the cell (giving a slope of 0.33°). The width of the channel was kept constant at 12cm. A sketch of the cell and an image of the foam within it are given in figure 1. The foaming solution was 5% Fairy liquid in deionised water. A fairly monodisperse foam (bubble volume $30.41 \pm 5.93$mm$^3$) was produced by blowing compressed air at a rate of $Q =$ 0.1 litres per minute (lpm) through a G18 needle (inner diameter 0.838mm). The flow-rate was then reduced to a lower value of $Q = 0.02$ lpm to slowly push the bubbles through the cell at a velocity of about 1mm/s. At these low velocities, we observe no deformation of the bubbles at the sides of the channel, and therefore presume that viscous effects are not significant in determining the transition point between 3D and 2D. High resolution video images ($1388 \times 1040$ pixels) of the region within the flowing foam where 3D to 2D transitions occurred were recorded at a rate of 2 fps.

Analysis of the images was carried out using ImageJ [7]. We tracked configurations in which two bubbles in the bulk of the foam were stacked directly on top of each other,

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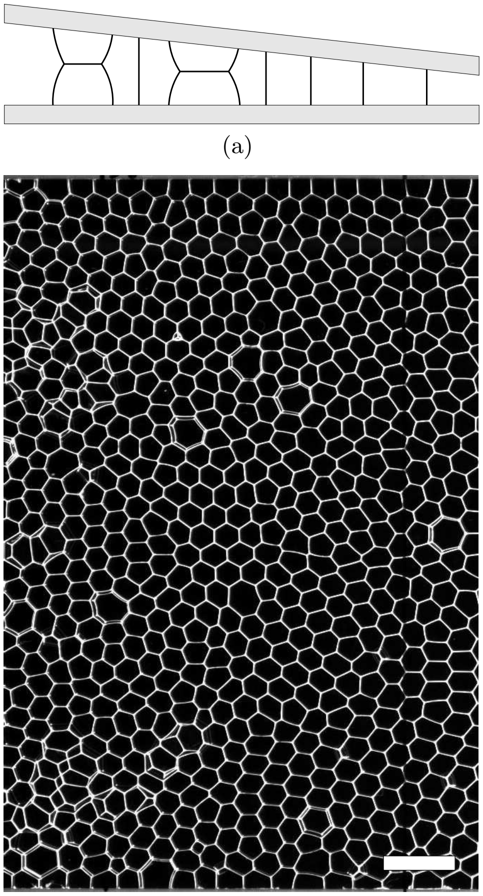

(a)

(b)

**Fig. 1.** Experiments, with flow from left to right: (a) Side view of the tapered Hele-Shaw cell used for the experiments, containing bubbles that touch both glass plates (a so-called 2D foam) and bubbles that don't (3D foam). (b) Typical image of the foam, taken from above, with soap films appearing white. The total width of the region shown is 78mm, and the scale bar has length 10mm.

which were easily recognisable due to the distinctive 'double wall' that is visible when observed from above (figure 1(b)). The number of sides ($N$) of the horizontal inter-bubble film between the two bubbles was recorded, as well as the plate separation at the point at which they underwent the transition to two "2D" bubbles. At this point the volume of each of the two bubbles was calculated from its cross-section and position in the cell. Figure 2 shows the average value of the critical height for each value of $N$, with the error bars giving the standard deviation of the range of values obtained. (Due to the small volumes and high uncertainty in the values for $N = 4$, these results are not shown.) We find that the critical separation decreases with $N$. Note that the values are lower than for the 2D to 3D instability described previously [3], suggesting that a foam must be compressed quite significantly to remove all two-layer configurations.

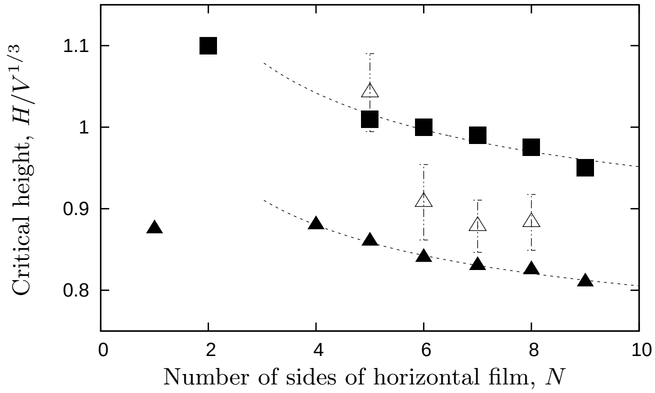


**Fig. 2.** Critical separation $H_c(N)/V^{1/3}$ for clusters that are both free (triangles) and adjacent to a wall (squares). Experimental values are shown with open triangles, simulation with filled symbols. The values for $N = 1$ (free) and $N = 2$ (wall) correspond to just two bubbles, without a surrounding cluster. The lines are fits to the simulation data for $N \geq 4$ in the form of eq. (1).

## 3 Simulations

Rather than considering an extensive foam with scattered double-layer bubbles as in the experiments described above, which would be computationally prohibitive, we simulate a finite cluster of bubbles in a two-layer structure. The structure that we use is based on the same "flower" cluster used previously [3], but now with two central bubbles, one on top of the other, surrounded by $N$ others that touch both bounding plates, as illustrated in figure 3(a). For simplicity we consider a monodisperse foam in which all bubbles have the same volume. We also explore the effects of the bubbles being in contact with a side-wall, since the presence of the planar wall bounding the cluster may change the critical separation – see figure 3(d).

We use the Surface Evolver [8] to predict numerically the critical separation for values of $N$ between 4 and 9. We construct clusters of bubbles with unit volumes (without loss of generality) with three levels of refinement of the triangulation, as shown in figure 3. Note that the wetting films on the glass plates play no rôle here and are omitted. The surface area of each cluster is reduced to a minimum, then the plate separation $H$ reduced by a small amount and the minimization repeated. For each value of $H$, we recorded the eigenvalues of the Hessian matrix of the surface area (energy) as a means to determine the value $H = H_c$ at which the eigenvalue associated with the film-tilting instability goes to zero [9]; the results are shown in figure 2. Due to the symmetry of the simulated cluster, the instability does not always appear spontaneously, but these values were confirmed by perturbing the cluster randomly at each step.

## 4 Discussion

Figure 2 shows that the critical separation in the simulations is very close to the values found in experiment; the

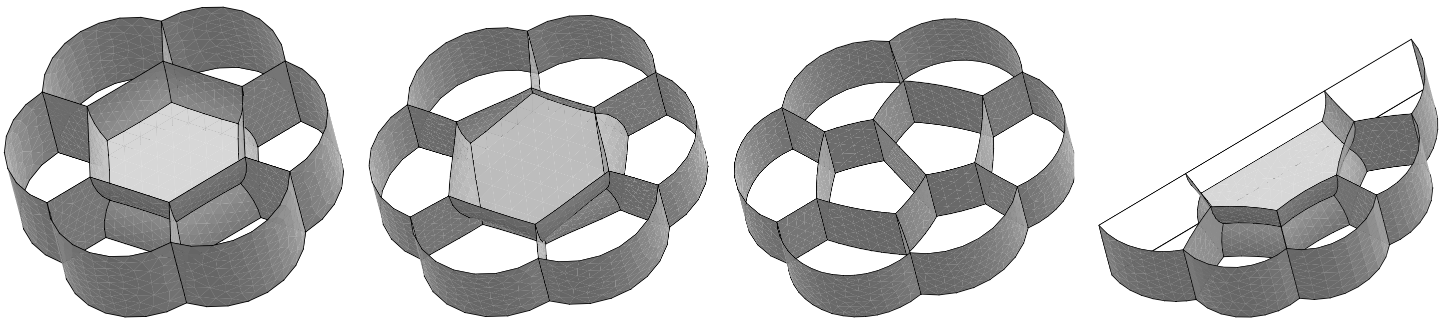


**Fig. 3.** Simulations: (a) $H/V^{1/3} = 1$: a pair of six-sided bubbles, one on top of the other, surrounded by six other bubbles. Films touching the bounding plates have been removed for clarity. (b) $H/V^{1/3} = 0.78$: the instability at $H_c(N = 6)/V^{1/3} = 0.81$ causes the central, horizontal film to tilt. (c) $H/V^{1/3} = 0.76$: even further below the critical separation there is a topological change and then all bubbles span the gap between the plates. In this case the hexagonal central bubbles become pentagonal. (d) $H/V^{1/3} = 1$: the case $N = 6$ (five bubbles and one wall) for a bubble cluster against a plane wall.

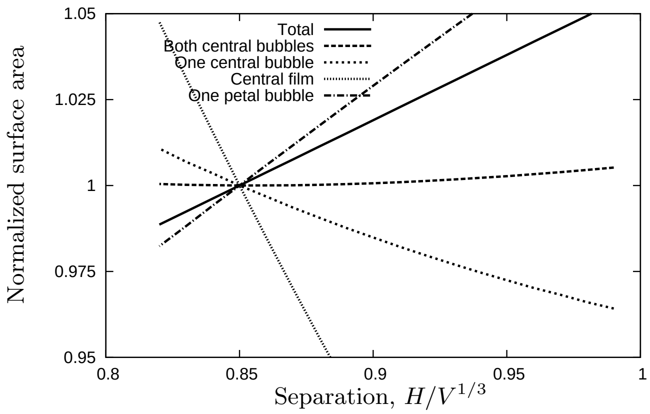


**Fig. 4.** The surface area of various elements of the bulk cluster, as the separation varies, in the case $N = 6$. Each area is normalized by its value at $H_c/V^{1/3} = 0.85$, the point at which the instability occurs. The minimum in the surface area of the two central bubbles indicates that it is the central bubbles that drive the instability.

discrepancy – that experimental values are slightly higher (that is, the instability is triggered earlier in an experiment in which $H$ is decreased) – can be attributed to the finite liquid fraction of the experiments. The critical separation decreases slightly with increasing $N$ and is predicted to be greater for a cluster adjacent to a wall.

It is clear [5] that the instability is driven by minimization of surface area. The simulations allow us to separate how the surface area of different parts of the cluster varies with $N$, shown in figure 4, and to show that only one curve, corresponding to the central bubbles and not the whole cluster, goes through a minimum to trigger the instability. That is, as the separation $H$ of the plates is decreased, the surface area of the two central bubbles decreases until it passes through a minimum. At this point the instability occurs (figure 3b) and the inter-bubble film tilts, since this now lowers the surface area of these two bubbles. As the separation decreases further, this causes a topological change and both bubbles then touch both bounding plates (figure 3c).

To predict the dependence of the critical separation on $N$, it is required to determine the surface area of the cluster analytically. However, the curvature of the soap films is significant and any analytic approximation to the area must take this into account. At the level of detail required, the calculations are too complex to give a simple functional dependence $H_c(N)$, and the simulations provide a much better idea of the area of the clusters. We show in figure 2 that a fit of the simulation data to the form

$$H_c/V^{1/3} = a_1 + a_2 \frac{1}{\sqrt{N}}, \tag{1}$$

with $a_1$ and $a_2$ of order one, provides an excellent approximation to the critical separation and suggests that $H_c$ decreases with $N^{-1/2}$.

## 5 Conclusions

We have described the transition which takes a 3D foam to a 2D one, and explained it with an argument based upon minimization of surface area. The values of the critical plate separation at which the instability occurs are lower than for transition which takes a 2D foam to a 3D one under extension (increasing $H$), described previously [3]. So a foam must be compressed quite significantly to remove all two-layer configurations.

The critical height depends weakly on the number of sides $N$ of a bubble, and the result [5] for two bubbles (shown for $N = 1$ on figure 2) provides a good approximation. For a given value of $N$, bubbles adjacent to a wall undergo the instability at higher $H$. Thus, in an experiment with fixed plate separation it will first become apparent that the bubble size is too large to sustain a two-layer foam close to the wall. Conversely, if the plate separation is being decreased, the transition to a 2D foam will start at the walls and propagate into the bulk.

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